NASA TECHNICAL NOTE **NASA TN D-3559**

NASA TN D-3559

ANALYSIS OF LUNAR AND SOLAR EFFECTS ON THE MOTION OF CLOSE EARTH SATELLITES

by James P. Murphy and Theodore L. Felsentreger

Goddard Space Flight Center
Greenbelt, Md.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • AUGUST 1966

NASA TN D-3559

ANALYSIS OF LUNAR AND SOLAR EFFECTS ON THE MOTION OF CLOSE EARTH SATELLITES

By James P. Murphy and Theodore L. Felsentreger

Goddard Space Flight Center
Greenbelt, Md.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – Price $2.50

ABSTRACT

Theoretical and experimental studies indicate that long periodic lunar and solar forces produce perturbations in the orbital elements of close earth satellites comparable to perturbations caused by the earth's oblateness.

Analytic formulas for secular and long-period luni-solar gravitational perturbations, as well as long-period expressions for solar radiation pressure effects (neglecting the earth's shadow), are given. A program for computing luni-solar perturbations was written and it was used to analyze the motion of the Relay 1 (1962 B, Upsilon 1) and Telstar 2 (1963 13A) satellites.

CONTENTS

ANALYSIS OF LUNAR AND SOLAR EFFECTS ON THE MOTION OF CLOSE EARTH SATELLITES

by

James P. Murphy and Theodore L. Felsentreger

Goddard Space Flight Center

INTRODUCTION

Most orbit theories for close earth satellites neglect lunar and solar forces with respect to the effects caused by the oblateness of the earth. However, an examination of the parameters in the long-period parts of the disturbing functions indicates that luni-solar forces give rise to perturbations which are comparable to long-period zonal harmonic effects.

Consequently, it is the purpose of this paper to present analytic formulas representing the major secular and long-period luni-solar perturbations (including solar radiation pressure) in the orbital elements of close earth satellites and to use them to study the motions of the Relay 1 and Telstar 2 satellites. Mathematical developments of the lunar and solar gravitational disturbing functions have been made by Kozai (Reference 1), Musen, Bailie, and Upton (Reference 2), and Kaula (Reference 3). In addition, Kaula (Reference 3) and Musen (Reference 4) have investigated the influence of solar radiation pressure on the motion of a satellite.

The formulas presented herein are the result of a straightforward integration of the variation equations for the Keplerian elements, using previous developments for the secular and long period parts of the disturbing functions. Expressions for the perturbations in mean anomaly, argument of perigee, and longitude of ascending node include interaction terms dependent upon both lunar and solar forces and the earth's oblateness.

The satellites Relay 1 and Telstar 2 were chosen for analysis principally because they are near enough to the earth to be considered "close" satellites, yet are far enough away so as to be relatively unaffected by drag (the semimajor axes for Relay 1 (1962 B, Upsilon 1) and Telstar 2 (1963 13A) are about 1.7 and 1.9 earth radii, respectively).

A list of the symbols used in this paper may be found in Appendix A.

LUNAR AND SOLAR GRAVITATIONAL DISTURBING FUNCTIONS

The disturbing function for the sun (or moon) is

$$R = \frac{Gm'}{r'} \left[\frac{r^2}{r'^2} P_2(s) + \frac{r^3}{r'^3} P_3(s) + \cdots \right] .$$

For the present purpose, only secular and long-period terms from the second and third Legendre polynomials (i.e., only those terms not dependent upon the mean anomaly of the satellite) are considered. In addition, terms from the third Legendre polynomial having the eccentricity of the disturbing body as a multiplier are neglected. The earth's equatorial plane is adopted as the fundamental plane.

The secular and long-period part of the disturbing function, then, is

$$R = n'^2 m' a^2 \left\{ \left[\left(1 + \frac{3}{2} e^2\right) P + \frac{15}{8} e^2 Q \right] - \frac{a}{a'} \left[\left(\frac{15}{64} e + \frac{45}{256} e^3 \right) S + \frac{175}{64} e^3 V \right] \right\},$$

where

$$P = \frac{1}{4}\left(1 - \frac{3}{2}\sin^2 i\right)\left(1 - \frac{3}{2}\sin^2 i'\right) [1 + 3e' \cos(\lambda' - \omega' - \Omega')]$$

$$+ \frac{3}{16} \sin 2i \sin 2i' \left[\cos(\Omega' - \Omega) + \frac{3}{2} e' \cos(\lambda' - \omega' - \Omega) + \frac{3}{2} e' \cos(\lambda' - \omega' - 2\Omega' + \Omega)\right]$$

$$+ \frac{3}{16} \sin^2 i \sin^2 i' \left[\cos(2\Omega' - 2\Omega) + \frac{3}{2} e' \cos(\lambda' - \omega' + \Omega' - 2\Omega) + \frac{3}{2} e' \cos(\lambda' - \omega' - 3\Omega' + 2\Omega)\right]$$

$$+ \frac{3}{8}\left(1 - \frac{3}{2}\sin^2 i\right) \sin^2 i' \left[\cos(2\lambda' - 2\Omega') - \frac{1}{2} e' \cos(\lambda' + \omega' - \Omega') + \frac{7}{2} e' \cos(3\lambda' - \omega' - 3\Omega')\right]$$

$$+ \frac{3}{8} \sin^2 i \cos^4 \frac{i'}{2} \left[\cos(2\lambda' - 2\Omega) - \frac{1}{2} e' \cos(\lambda' + \omega' + \Omega' - 2\Omega) + \frac{7}{2} e' \cos(3\lambda' - \omega' - \Omega' - 2\Omega)\right]$$

$$- \frac{3}{8} \sin 2i \sin i' \cos^2 \frac{i'}{2} \left[\cos(2\lambda' - \Omega' - \Omega) - \frac{1}{2} e' \cos(\lambda' + \omega' - \Omega) + \frac{7}{2} e' \cos(3\lambda' - \omega' - 2\Omega' - \Omega)\right]$$

$$+ \frac{3}{8} \sin^2 i \sin^4 \frac{i'}{2} \left[\cos(2\lambda' - 4\Omega' + 2\Omega) - \frac{1}{2} e' \cos(\lambda' + \omega' - 3\Omega' + 2\Omega) + \frac{7}{2} e' \cos(3\lambda' - \omega' - 5\Omega' + 2\Omega)\right]$$

$$+ \frac{3}{8} \sin 2i \sin i' \sin^2 \frac{i'}{2} \left[\cos(2\lambda' - 3\Omega' + \Omega) - \frac{1}{2} e' \cos(\lambda' + \omega' - 2\Omega' + \Omega) + \frac{7}{2} e' \cos(3\lambda' - \omega' - 4\Omega' + \Omega)\right],$$

$$Q = \cos^4\frac{i}{2}\cos^4\frac{i'}{2}\left[\cos(2\lambda' - 2\omega - 2\Omega) - \frac{1}{2}e'\cos(\lambda' + \omega' + \Omega' - 2\omega - 2\Omega) + \frac{7}{2}e'\cos(3\lambda' - \omega' - \Omega' - 2\omega - 2\Omega)\right]$$

$$+ \frac{1}{2}\sin^2 i\left(1 - \frac{3}{2}\sin^2 i'\right)\left[\cos 2\omega + \frac{3}{2}e'\cos(\lambda' - \omega' - \Omega' - 2\omega) + \frac{3}{2}e'\cos(\lambda' - \omega' - \Omega' + 2\omega)\right]$$

$$+ \frac{1}{2}\cos^4\frac{i}{2}\sin^2 i'\left[\cos(2\Omega' - 2\omega - 2\Omega) + \frac{3}{2}e'\cos(\lambda' - \omega' + \Omega' - 2\omega - 2\Omega) + \frac{3}{2}e'\cos(\lambda' - \omega' - 3\Omega' + 2\omega + 2\Omega)\right]$$

$$+ \sin^4\frac{i}{2}\cos^4\frac{i'}{2}\left[\cos(2\lambda' + 2\omega - 2\Omega) - \frac{1}{2}e'\cos(\lambda' + \omega' + \Omega' + 2\omega - 2\Omega) + \frac{7}{2}e'\cos(3\lambda' - \omega' - \Omega' + 2\omega - 2\Omega)\right]$$

$$+ \cos^4\frac{i}{2}\sin^4\frac{i'}{2}\left[\cos(2\lambda' - 4\Omega' + 2\omega + 2\Omega) - \frac{1}{2}e'\cos(\lambda' + \omega' - 3\Omega' + 2\omega + 2\Omega) + \frac{7}{2}e'\cos(3\lambda' - \omega' - 5\Omega' + 2\omega + 2\Omega)\right]$$

$$+ \sin^4\frac{i}{2}\sin^4\frac{i'}{2}\left[\cos(2\lambda' - 4\Omega' - 2\omega + 2\Omega) - \frac{1}{2}e'\cos(\lambda' + \omega' - 3\Omega' - 2\omega + 2\Omega) + \frac{7}{2}e'\cos(3\lambda' - \omega' - 5\Omega' - 2\omega + 2\Omega)\right]$$

$$+ \frac{3}{8}\sin^2 i\sin^2 i'\left[\cos(2\lambda' - 2\Omega' - 2\omega) - \frac{1}{2}e'\cos(\lambda' + \omega' - \Omega' - 2\omega) + \frac{7}{2}e'\cos(3\lambda' - \omega' - 3\Omega' - 2\omega)\right]$$

$$+ \frac{3}{8}\sin^2 i\sin^2 i'\left[\cos(2\lambda' - 2\Omega' + 2\omega) - \frac{1}{2}e'\cos(\lambda' + \omega' - \Omega' + 2\omega) + \frac{7}{2}e'\cos(3\lambda' - \omega' - 3\Omega' + 2\omega)\right]$$

$$+ \sin i\cos^2\frac{i}{2}\sin i'\cos^2\frac{i'}{2}\left[\cos(2\lambda' - \Omega' - 2\omega - \Omega) - \frac{1}{2}e'\cos(\lambda' + \omega' - 2\omega - \Omega) + \frac{7}{2}e'\cos(3\lambda' - \omega' - 2\Omega' - 2\omega - \Omega)\right]$$

$$- \frac{1}{2}\sin i\cos^2\frac{i}{2}\sin 2i'\left[\cos(\Omega' - 2\omega - \Omega) + \frac{3}{2}e'\cos(\lambda' - \omega' - 2\omega - \Omega) + \frac{3}{2}e'\cos(\lambda' - \omega' - 2\Omega' + 2\omega + \Omega)\right]$$

$$+ \frac{1}{2}\sin^4\frac{i}{2}\sin^2 i'\left[\cos(2\Omega' + 2\omega - 2\Omega) + \frac{3}{2}e'\cos(\lambda' - \omega' - 3\Omega' - 2\omega + 2\Omega) + \frac{3}{2}e'\cos(\lambda' - \omega' + \Omega' + 2\omega - 2\Omega)\right]$$

$$+ \frac{1}{2}\sin i\sin^2\frac{i}{2}\sin 2i'\left[\cos(\Omega' + 2\omega - \Omega) + \frac{3}{2}e'\cos(\lambda' - \omega' - 2\Omega' - 2\omega + \Omega) + \frac{3}{2}e'\cos(\lambda' - \omega' + 2\omega - \Omega)\right]$$

$$- \sin i\sin^2\frac{i}{2}\sin i'\cos^2\frac{i'}{2}\left[\cos(2\lambda' - \Omega' + 2\omega - \Omega) - \frac{1}{2}e'\cos(\lambda' + \omega' + 2\omega - \Omega) + \frac{7}{2}e'\cos(3\lambda' - \omega' - 2\Omega' + 2\omega - \Omega)\right]$$

$$- \sin i\cos^2\frac{i}{2}\sin i'\sin^2\frac{i'}{2}\left[\cos(2\lambda' - 3\Omega' + 2\omega + \Omega) - \frac{1}{2}e'\cos(\lambda' + \omega' - 2\Omega' + 2\omega + \Omega) + \frac{7}{2}e'\cos(3\lambda' - \omega' - 4\Omega' + 2\omega + \Omega)\right]$$

$$+ \sin i\sin^2\frac{i}{2}\sin i'\sin^2\frac{i'}{2}\left[\cos(2\lambda' - 3\Omega' - 2\omega + \Omega) - \frac{1}{2}e'\cos(\lambda' + \omega' - 2\Omega' - 2\omega + \Omega) + \frac{7}{2}e'\cos(3\lambda' - \omega' - 4\Omega' - 2\omega + \Omega)\right],$$

$$
\begin{aligned}
S = & -\frac{3}{4}\sin i\left(1-5\cos^2 i\right)\sin i'\left(1-5\cos^2 i'\right)\cos\left(\lambda'-\Omega'+\omega\right) \\
& +\frac{1}{4}\cos^2\frac{i}{2}\left(1+10\cos i-15\cos^2 i\right)\sin^2\frac{i'}{2}\left(1-10\cos i'-15\cos^2 i'\right)\cos\left(\lambda'-2\Omega'+\omega+\Omega\right) \\
& +\frac{1}{4}\sin^2\frac{i}{2}\left(1-10\cos i-15\cos^2 i\right)\cos^2\frac{i'}{2}\left(1+10\cos i'-15\cos^2 i'\right)\cos\left(\lambda'+\omega-\Omega\right) \\
& -\frac{5}{8}\sin i\left(1-2\cos i-3\cos^2 i\right)\sin i'\left(1+2\cos i'-3\cos^2 i'\right)\cos\left(\lambda'-3\Omega'+\omega+2\Omega\right) \\
& -\frac{5}{8}\sin i\left(1+2\cos i-3\cos^2 i\right)\sin i'\left(1-2\cos i'-3\cos^2 i'\right)\cos\left(\lambda'+\Omega'+\omega-2\Omega\right) \\
& +\frac{15}{4}\sin^2 i\cos^2\frac{i}{2}\sin^2 i'\sin^2\frac{i'}{2}\cos\left(\lambda'-4\Omega'+\omega+3\Omega\right) \\
& +\frac{15}{4}\sin^2 i\sin^2\frac{i}{2}\sin^2 i'\cos^2\frac{i'}{2}\cos\left(\lambda'+2\Omega'+\omega-3\Omega\right) \\
& +\frac{3}{4}\sin i\left(1-5\cos^2 i\right)\sin i'\left(1-5\cos^2 i'\right)\cos\left(\lambda'-\Omega'-\omega\right) \\
& +\frac{1}{4}\cos^2\frac{i}{2}\left(1+10\cos i-15\cos^2 i\right)\cos^2\frac{i'}{2}\left(1+10\cos i'-15\cos^2 i'\right)\cos\left(\lambda'-\omega-\Omega\right) \\
& +\frac{1}{4}\sin^2\frac{i}{2}\left(1-10\cos i-15\cos^2 i\right)\sin^2\frac{i'}{2}\left(1-10\cos i'-15\cos^2 i'\right)\cos\left(\lambda'-2\Omega'-\omega+\Omega\right) \\
& +\frac{5}{8}\sin i\left(1-2\cos i-3\cos^2 i\right)\sin i'\left(1-2\cos i'-3\cos^2 i'\right)\cos\left(\lambda'+\Omega'-\omega-2\Omega\right) \\
& +\frac{5}{8}\sin i\left(1+2\cos i-3\cos^2 i\right)\sin i'\left(1+2\cos i'-3\cos^2 i'\right)\cos\left(\lambda'-3\Omega'-\omega+2\Omega\right) \\
& +\frac{15}{4}\sin^2 i\cos^2\frac{i}{2}\sin^2 i'\cos^2\frac{i'}{2}\cos\left(\lambda'+2\Omega'-\omega-3\Omega\right) \\
& +\frac{15}{4}\sin^2 i\sin^2\frac{i}{2}\sin^2 i'\sin^2\frac{i'}{2}\cos\left(\lambda'-4\Omega'-\omega+3\Omega\right)
\end{aligned}
$$

$$+ \frac{5}{4} \sin i \left(1 - 5\cos^2 i\right) \sin^3 i' \cos(3\lambda' - 3\Omega' + \omega)$$

$$- \frac{5}{4} \cos^2 \frac{i}{2} \left(1 + 10\cos i - 15\cos^2 i\right) \sin^2 i' \sin^2 \frac{i'}{2} \cos(3\lambda' - 4\Omega' + \omega + \Omega)$$

$$- \frac{5}{4} \sin^2 \frac{i}{2} \left(1 - 10\cos i - 15\cos^2 i\right) \sin^2 i' \cos^2 \frac{i'}{2} \cos(3\lambda' - 2\Omega' + \omega - \Omega)$$

$$- \frac{5}{2} \sin i \left(1 - 2\cos i - 3\cos^2 i\right) \sin i' \sin^4 \frac{i'}{2} \cos(3\lambda' - 5\Omega' + \omega + 2\Omega)$$

$$- \frac{5}{2} \sin i \left(1 + 2\cos i - 3\cos^2 i\right) \sin i' \cos^4 \frac{i'}{2} \cos(3\lambda' - \Omega' + \omega - 2\Omega)$$

$$+ 5 \sin^2 i \cos^2 \frac{i}{2} \sin^6 \frac{i'}{2} \cos(3\lambda' - 6\Omega' + \omega + 3\Omega)$$

$$+ 5 \sin^2 i \sin^2 \frac{i}{2} \cos^6 \frac{i'}{2} \cos(3\lambda' + \omega - 3\Omega)$$

$$- \frac{5}{4} \sin i \left(1 - 5\cos^2 i\right) \sin^3 i' \cos(3\lambda' - 3\Omega' - \omega)$$

$$- \frac{5}{4} \cos^2 \frac{i}{2} \left(1 + 10\cos i - 15\cos^2 i\right) \sin^2 i' \cos^2 \frac{i'}{2} \cos(3\lambda' - 2\Omega' - \omega - \Omega)$$

$$- \frac{5}{4} \sin^2 \frac{i}{2} \left(1 - 10\cos i - 15\cos^2 i\right) \sin^2 i' \sin^2 \frac{i'}{2} \cos(3\lambda' - 4\Omega' - \omega + \Omega)$$

$$+ \frac{5}{2} \sin i \left(1 - 2\cos i - 3\cos^2 i\right) \sin i' \cos^4 \frac{i'}{2} \cos(3\lambda' - \Omega' - \omega - 2\Omega)$$

$$+ \frac{5}{2} \sin i \left(1 + 2\cos i - 3\cos^2 i\right) \sin i' \sin^4 \frac{i'}{2} \cos(3\lambda' - 5\Omega' - \omega + 2\Omega)$$

$$+ 5 \sin^2 i \cos^2 \frac{i}{2} \cos^6 \frac{i'}{2} \cos(3\lambda' - \omega - 3\Omega)$$

$$+ 5 \sin^2 i \sin^2 \frac{i}{2} \sin^6 \frac{i'}{2} \cos(3\lambda' - 6\Omega' - \omega + 3\Omega) ,$$

and

$$
\begin{aligned}
V = & \frac{3}{16} \sin^3 i \sin i' \left(1 - 5\cos^2 i'\right) \cos(\lambda' - \Omega' + 3\omega) \\
& - \frac{3}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 \frac{i'}{2} \left(1 - 10\cos i' - 15\cos^2 i'\right) \cos(\lambda' - 2\Omega' + 3\omega + \Omega) \\
& - \frac{3}{16} \sin^2 i \sin^2 \frac{i}{2} \cos^2 \frac{i'}{2} \left(1 + 10\cos i' - 15\cos^2 i'\right) \cos(\lambda' + 3\omega - \Omega) \\
& - \frac{3}{8} \sin i \cos^4 \frac{i}{2} \sin i' \left(1 + 2\cos i' - 3\cos^2 i'\right) \cos(\lambda' - 3\Omega' + 3\omega + 2\Omega) \\
& - \frac{3}{8} \sin i \sin^4 \frac{i}{2} \sin i' \left(1 - 2\cos i' - 3\cos^2 i'\right) \cos(\lambda' + \Omega' + 3\omega - 2\Omega) \\
& + \frac{3}{4} \cos^6 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \cos(\lambda' - 4\Omega' + 3\omega + 3\Omega) \\
& + \frac{3}{4} \sin^6 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \cos(\lambda' + 2\Omega' + 3\omega - 3\Omega) \\
& - \frac{3}{16} \sin^3 i \sin i' \left(1 - 5\cos^2 i'\right) \cos(\lambda' - \Omega' - 3\omega) \\
& - \frac{3}{16} \sin^2 i \cos^2 \frac{i}{2} \cos^2 \frac{i'}{2} \left(1 + 10\cos i' - 15\cos^2 i'\right) \cos(\lambda' - 3\omega - \Omega) \\
& - \frac{3}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 \frac{i'}{2} \left(1 - 10\cos i' - 15\cos^2 i'\right) \cos(\lambda' - 2\Omega' - 3\omega + \Omega \\
& + \frac{3}{8} \sin i \cos^4 \frac{i}{2} \sin i' \left(1 - 2\cos i' - 3\cos^2 i'\right) \cos(\lambda' + \Omega' - 3\omega - 2\Omega) \\
& + \frac{3}{8} \sin i \sin^4 \frac{i}{2} \sin i' \left(1 + 2\cos i' - 3\cos^2 i'\right) \cos(\lambda' - 3\Omega' - 3\omega + 2\Omega) \\
& + \frac{3}{4} \cos^6 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \cos(\lambda' + 2\Omega' - 3\omega - 3\Omega) \\
& + \frac{3}{4} \sin^6 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \cos(\lambda' - 4\Omega' - 3\omega + 3\Omega)
\end{aligned}
$$

$$- \frac{5}{16} \sin^3 i \sin^3 i' \cos (3\lambda' - 3\Omega' + 3\omega)$$

$$+ \frac{15}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \cos (3\lambda' - 4\Omega' + 3\omega + \Omega)$$

$$+ \frac{15}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \cos (3\lambda' - 2\Omega' + 3\omega - \Omega)$$

$$- \frac{3}{2} \sin i \cos^4 \frac{i}{2} \sin i' \sin^4 \frac{i'}{2} \cos (3\lambda' - 5\Omega' + 3\omega + 2\Omega)$$

$$- \frac{3}{2} \sin i \sin^4 \frac{i}{2} \sin i' \cos^4 \frac{i'}{2} \cos (3\lambda' - \Omega' + 3\omega - 2\Omega)$$

$$+ \cos^6 \frac{i}{2} \sin^6 \frac{i'}{2} \cos (3\lambda' - 6\Omega' + 3\omega + 3\Omega)$$

$$+ \sin^6 \frac{i}{2} \cos^6 \frac{i'}{2} \cos (3\lambda' + 3\omega - 3\Omega)$$

$$+ \frac{5}{16} \sin^3 i \sin^3 i' \cos (3\lambda' - 3\Omega' - 3\omega)$$

$$+ \frac{15}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \cos (3\lambda' - 2\Omega' - 3\omega - \Omega)$$

$$+ \frac{15}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \cos (3\lambda' - 4\Omega' - 3\omega + \Omega)$$

$$+ \frac{3}{2} \sin i \cos^4 \frac{i}{2} \sin i' \cos^4 \frac{i'}{2} \cos (3\lambda' - \Omega' - 3\omega - 2\Omega)$$

$$+ \frac{3}{2} \sin i \sin^4 \frac{i}{2} \sin i' \sin^4 \frac{i'}{2} \cos (3\lambda' - 5\Omega' - 3\omega + 2\Omega)$$

$$+ \cos^6 \frac{i}{2} \cos^6 \frac{i'}{2} \cos (3\lambda' - 3\omega - 3\Omega)$$

$$+ \sin^6 \frac{i}{2} \sin^6 \frac{i'}{2} \cos (3\lambda' - 6\Omega' - 3\omega + 3\Omega) \quad .$$

PERTURBATIONS IN ORBITAL ELEMENTS

The disturbing function R is substituted into the variation equations

$$\left.\begin{aligned} \frac{de}{dt} &= -\frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \omega} \\ &\text{and} \\ \frac{di}{dt} &= \frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \omega} - \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \Omega} \end{aligned}\right\} \qquad (1)$$

and into

$$\left.\begin{aligned} \frac{d\ell}{dt} &= -\frac{2}{na}\frac{\partial R}{\partial a} - \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial e} + \frac{d\dot{\ell}}{de}\delta e + \frac{d\dot{\ell}}{di}\delta i , \\ \frac{d\omega}{dt} &= -\frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i} + \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial e} + \frac{d\dot{\omega}}{de}\delta e + \frac{d\dot{\omega}}{di}\delta i , \\ &\text{and} \\ \frac{d\Omega}{dt} &= \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i} + \frac{d\dot{\Omega}}{de}\delta e + \frac{d\dot{\Omega}}{di}\delta i . \end{aligned}\right\} \qquad (2)$$

In Equations 2, δe and δi are the perturbations obtained by integration of Equations 1, and $\dot{\ell}$, $\dot{\omega}$, and $\dot{\Omega}$ are given by (see Reference 5)

$$\left.\begin{aligned} \dot{\ell} = n \Bigg\{ & 1 - \frac{3J_2 a_e^2}{4a^2(1-e^2)^{3/2}}\left(1 - 3\cos^2 i\right) + \frac{3J_2^2 a_e^4}{128a^4(1-e^2)^{7/2}}\Big[10 - 25e^2 + 16\sqrt{1-e^2} \\ & - 6\left(10 - 15e^2 + 16\sqrt{1-e^2}\right)\cos^2 i + \left(130 - 25e^2 + 144\sqrt{1-e^2}\right)\cos^4 i\Big] \\ & - \frac{45 J_4 a_e^4 e^2}{128a^4(1-e^2)^{7/2}}\left(3 - 30\cos^2 i + 35\cos^4 i\right)\Bigg\} , \\ \dot{\omega} = n \Bigg\{ & -\frac{3J_2 a_e^2}{4a^2(1-e^2)^2}\left(1 - 5\cos^2 i\right) + \frac{3J_2^2 a_e^4}{128a^4(1-e^2)^4}\Big[-10 - 25e^2 + 24\sqrt{1-e^2} \\ & - 6\left(6 - 21e^2 + 32\sqrt{1-e^2}\right)\cos^2 i + 5\left(86 - 9e^2 + 72\sqrt{1-e^2}\right)\cos^4 i\Big] \\ & - \frac{15 J_4 a_e^4}{128a^4(1-e^2)^4}\Big[3\left(4 + 3e^2\right) - 18\left(8 + 7e^2\right)\cos^2 i + 7\left(28 + 27e^2\right)\cos^4 i\Big]\Bigg\} , \end{aligned}\right\} \qquad (3)$$

and

$$\left.\begin{aligned}\dot{\Omega} = n\Biggl\{&-\frac{3J_2 a_e^2 \cos i}{2a^2(1-e^2)^2} + \frac{3J_2^2 a_e^4 \cos i}{32a^4(1-e^2)^4}\left[4 - 9e^2 + 12\sqrt{1-e^2} - \left(40 - 5e^2 + 36\sqrt{1-e^2}\right)\cos^2 i\right]\\ &-\frac{15J_4 a_e^4 \left(2+3e^2\right)\cos i}{32a^4\left(1-e^2\right)^4}\left(3 - 7\cos^2 i\right)\Biggr\} .\end{aligned}\right\} \quad (3)$$

Integration of Equations 1 and 2 is accomplished by assuming that a, e, i, a', e', and i' are constants and that ω, Ω, λ', ω', and Ω' are linear functions of time. The secular motions for ω and Ω expressed by Equations 3 are used. Thus,

$$\delta e = -\frac{\sqrt{1-e^2}}{na^2 e} A_1$$

and

$$\delta i = \frac{\cos i}{na^2\sqrt{1-e^2}\sin i} A_1 - \frac{1}{na^2\sqrt{1-e^2}\sin i} A_2 , \quad (4)$$

and

$$\delta \ell = -\frac{2}{na} A_3 - \frac{1-e^2}{na^2 e} A_4 + \frac{d\dot{\ell}}{de} A_6 + \frac{d\dot{\ell}}{di} A_7 ,$$

$$\delta\omega = -\frac{\cos i}{na^2\sqrt{1-e^2}\sin i} A_5 + \frac{\sqrt{1-e^2}}{na^2 e} A_4 + \frac{d\dot{\omega}}{de} A_6 + \frac{d\dot{\omega}}{di} A_7 ,$$

and

$$\delta\Omega = \frac{1}{na^2\sqrt{1-e^2}\sin i} A_5 + \frac{d\dot{\Omega}}{de} A_6 + \frac{d\dot{\Omega}}{di} A_7 . \quad (5)$$

Expressions for the various A_i are as follows:

$$A_1 = \int \frac{\partial R}{\partial \omega}\, dt$$

$$= -\frac{15}{64} n'^2 m' a^2 e^2 \Bigg\{ 8 \cos^4 \frac{i}{2} \cos^4 \frac{i'}{2} \left[\frac{2 \cos (2\lambda' - 2\omega - 2\Omega)}{2(\dot\lambda' - \dot\omega - \dot\Omega)} - e' \frac{\cos (\lambda' + \omega' + \Omega' - 2\omega - 2\Omega)}{\dot\lambda' + \dot\omega' + \dot\Omega' - 2\dot\omega - 2\dot\Omega} + 7 e' \frac{\cos (3\lambda' - \omega' - \Omega' - 2\omega - 2\Omega)}{3\dot\lambda' - \dot\omega' - \dot\Omega' - 2\dot\omega - 2\dot\Omega} \right]$$

$$- 4 \sin^2 i \left(1 - \frac{3}{2} \sin^2 i'\right) \left[\frac{2 \cos 2\omega}{2\dot\omega} - 3e' \frac{\cos (\lambda' - \omega' - \Omega' - 2\omega)}{\dot\lambda' - \dot\omega' - \dot\Omega' - 2\dot\omega} + 3e' \frac{\cos (\lambda' - \omega' - \Omega' + 2\omega)}{\dot\lambda' - \dot\omega' - \dot\Omega' + 2\dot\omega} \right]$$

$$+ 4 \cos^4 \frac{i}{2} \sin^2 i' \left[\frac{2 \cos (2\Omega' - 2\omega - 2\Omega)}{2(\dot\Omega' - \dot\omega - \dot\Omega)} + 3e' \frac{\cos (\lambda' - \omega' + \Omega' - 2\omega - 2\Omega)}{\dot\lambda' - \dot\omega' + \dot\Omega' - 2\dot\omega - 2\dot\Omega} - 3e' \frac{\cos (\lambda' - \omega' - 3\Omega' + 2\omega + 2\Omega)}{\dot\lambda' - \dot\omega' - 3\dot\Omega' + 2\dot\omega + 2\dot\Omega} \right]$$

$$- 8 \sin^4 \frac{i}{2} \cos^4 \frac{i'}{2} \left[\frac{2 \cos (2\lambda' + 2\omega - 2\Omega)}{2(\dot\lambda' + \dot\omega - \dot\Omega)} - e' \frac{\cos (\lambda' + \omega' + \Omega' + 2\omega - 2\Omega)}{\dot\lambda' + \dot\omega' + \dot\Omega' + 2\dot\omega - 2\dot\Omega} + 7e' \frac{\cos (3\lambda' - \omega' - \Omega' + 2\omega - 2\Omega)}{3\dot\lambda' - \dot\omega' - \dot\Omega' + 2\dot\omega - 2\dot\Omega} \right]$$

$$- 8 \cos^4 \frac{i}{2} \sin^4 \frac{i'}{2} \left[\frac{2 \cos (2\lambda' - 4\Omega' + 2\omega + 2\Omega)}{2(\dot\lambda' - 2\dot\Omega' + \dot\omega + \dot\Omega)} - e' \frac{\cos (\lambda' + \omega' - 3\Omega' + 2\omega + 2\Omega)}{\dot\lambda' + \dot\omega' - 3\dot\Omega' + 2\dot\omega + 2\dot\Omega} + 7e' \frac{\cos (3\lambda' - \omega' - 5\Omega' + 2\omega + 2\Omega)}{3\dot\lambda' - \dot\omega' - 5\dot\Omega' + 2\dot\omega + 2\dot\Omega} \right]$$

$$+ 8 \sin^4 \frac{i}{2} \sin^4 \frac{i'}{2} \left[\frac{2 \cos (2\lambda' - 4\Omega' - 2\omega + 2\Omega)}{2(\dot\lambda' - 2\dot\Omega' - \dot\omega + \dot\Omega)} - e' \frac{\cos (\lambda' + \omega' - 3\Omega' - 2\omega + 2\Omega)}{\dot\lambda' + \dot\omega' - 3\dot\Omega' - 2\dot\omega + 2\dot\Omega} + 7e' \frac{\cos (3\lambda' - \omega' - 5\Omega' - 2\omega + 2\Omega)}{3\dot\lambda' - \dot\omega' - 5\dot\Omega' - 2\dot\omega + 2\dot\Omega} \right]$$

$$+ 3 \sin^2 i \sin^2 i' \left[\frac{2 \cos (2\lambda' - 2\Omega' - 2\omega)}{2(\dot\lambda' - \dot\Omega' - \dot\omega)} - e' \frac{\cos (\lambda' + \omega' - \Omega' - 2\omega)}{\dot\lambda' + \dot\omega' - \dot\Omega' - 2\dot\omega} + 7e' \frac{\cos (3\lambda' - \omega' - 3\Omega' - 2\omega)}{3\dot\lambda' - \dot\omega' - 3\dot\Omega' - 2\dot\omega} \right]$$

$$- 3 \sin^2 i \sin^2 i' \left[\frac{2 \cos (2\lambda' - 2\Omega' + 2\omega)}{2(\dot\lambda' - \dot\Omega' + \dot\omega)} - e' \frac{\cos (\lambda' + \omega' - \Omega' + 2\omega)}{\dot\lambda' + \dot\omega' - \dot\Omega' + 2\dot\omega} + 7e' \frac{\cos (3\lambda' - \omega' - 3\Omega' + 2\omega)}{3\dot\lambda' - \dot\omega' - 3\dot\Omega' + 2\dot\omega} \right]$$

$$+ 8 \sin i \cos^2 \frac{i}{2} \sin i' \cos^2 \frac{i'}{2} \left[2 \frac{\cos (2\lambda' - \Omega' - 2\omega - \Omega)}{2\dot\lambda' - \dot\Omega' - 2\dot\omega - \dot\Omega} - e' \frac{\cos (\lambda' + \omega' - 2\omega - \Omega)}{\dot\lambda' + \dot\omega' - 2\dot\omega - \dot\Omega} + 7e' \frac{\cos (3\lambda' - \omega' - 2\Omega' - 2\omega - \Omega)}{3\dot\lambda' - \dot\omega' - 2\dot\Omega' - 2\dot\omega - \dot\Omega} \right]$$

$$- 4 \sin i \cos^2 \frac{i}{2} \sin 2i' \left[2 \frac{\cos (\Omega' - 2\omega - \Omega)}{\dot\Omega' - 2\dot\omega - \dot\Omega} + 3e' \frac{\cos (\lambda' - \omega' - 2\omega - \Omega)}{\dot\lambda' - \dot\omega' - 2\dot\omega - \dot\Omega} - 3e' \frac{\cos (\lambda' - \omega' - 2\Omega' + 2\omega + \Omega)}{\dot\lambda' - \dot\omega' - 2\dot\Omega' + 2\dot\omega + \dot\Omega} \right]$$

$$- 4 \sin^4 \frac{i}{2} \sin^2 i' \left[\frac{2 \cos (2\Omega' + 2\omega - 2\Omega)}{2(\dot{\Omega}' + \dot{\omega} - \dot{\Omega})} - 3e' \frac{\cos (\lambda' - \omega' - 3\Omega' - 2\omega + 2\Omega)}{\dot{\lambda}' - \dot{\omega}' - 3\dot{\Omega}' - 2\dot{\omega} + 2\dot{\Omega}} + 3e' \frac{\cos (\lambda' - \omega' + \Omega' + 2\omega - 2\Omega)}{\dot{\lambda}' - \dot{\omega}' + \dot{\Omega}' + 2\dot{\omega} - 2\dot{\Omega}} \right]$$

$$- 4 \sin i \sin^2 \frac{i}{2} \sin 2i' \left[2 \frac{\cos (\Omega' + 2\omega - \Omega)}{\dot{\Omega}' + 2\dot{\omega} - \dot{\Omega}} - 3e' \frac{\cos (\lambda' - \omega' - 2\Omega' - 2\omega + \Omega)}{\dot{\lambda}' - \dot{\omega}' - 2\dot{\Omega}' - 2\dot{\omega} + \dot{\Omega}} + 3e' \frac{\cos (\lambda' - \omega' + 2\omega - \Omega)}{\dot{\lambda}' - \dot{\omega}' + 2\dot{\omega} - \dot{\Omega}} \right]$$

$$+ 8 \sin i \sin^2 \frac{i}{2} \sin i' \cos^2 \frac{i'}{2} \left[2 \frac{\cos (2\lambda' - \Omega' + 2\omega - \Omega)}{2\dot{\lambda}' - \dot{\Omega}' + 2\dot{\omega} - \dot{\Omega}} - e' \frac{\cos (\lambda' + \omega' + 2\omega - \Omega)}{\dot{\lambda}' + \dot{\omega}' + 2\dot{\omega} - \dot{\Omega}} + 7e' \frac{\cos (3\lambda' - \omega' - 2\Omega' + 2\omega - \Omega)}{3\dot{\lambda}' - \dot{\omega}' - 2\dot{\Omega}' + 2\dot{\omega} - \dot{\Omega}} \right]$$

$$+ 8 \sin i \cos^2 \frac{i}{2} \sin i' \sin^2 \frac{i'}{2} \left[2 \frac{\cos (2\lambda' - 3\Omega' + 2\omega + \Omega)}{2\dot{\lambda}' - 3\dot{\Omega}' + 2\dot{\omega} + \dot{\Omega}} - e' \frac{\cos (\lambda' + \omega' - 2\Omega' + 2\omega + \Omega)}{\dot{\lambda}' + \dot{\omega}' - 2\dot{\Omega}' + 2\dot{\omega} + \dot{\Omega}} + 7e' \frac{\cos (3\lambda' - \omega' - 4\Omega' + 2\omega + \Omega)}{3\dot{\lambda}' - \dot{\omega}' - 4\dot{\Omega}' + 2\dot{\omega} + \dot{\Omega}} \right]$$

$$+ 8 \sin i \sin^2 \frac{i}{2} \sin i' \sin^2 \frac{i'}{2} \left[2 \frac{\cos (2\lambda' - 3\Omega' - 2\omega + \Omega)}{2\dot{\lambda}' - 3\dot{\Omega}' - 2\dot{\omega} + \dot{\Omega}} - e' \frac{\cos (\lambda' + \omega' - 2\Omega' - 2\omega + \Omega)}{\dot{\lambda}' + \dot{\omega}' - 2\dot{\Omega}' - 2\dot{\omega} + \dot{\Omega}} + 7e' \frac{\cos (3\lambda' - \omega' - 4\Omega' - 2\omega + \Omega)}{3\dot{\lambda}' - \dot{\omega}' - 4\dot{\Omega}' - 2\dot{\omega} + \dot{\Omega}} \right] \Bigg\}$$

$$- m' \frac{a^3}{a'^4} \left(\frac{15}{64} e + \frac{45}{256} e^3 \right) \Bigg[- \frac{3}{4} \sin i \left(1 - 5 \cos^2 i\right) \sin i' \left(1 - 5 \cos^2 i'\right) \frac{\cos (\lambda' - \Omega' + \omega)}{\dot{\lambda}' - \dot{\Omega}' + \dot{\omega}}$$

$$+ \frac{1}{4} \cos^2 \frac{i}{2} \left(1 + 10 \cos i - 15 \cos^2 i\right) \sin^2 \frac{i'}{2} \left(1 - 10 \cos i' - 15 \cos^2 i'\right) \frac{\cos (\lambda' - 2\Omega' + \omega + \Omega)}{\dot{\lambda}' - 2\dot{\Omega}' + \dot{\omega} + \dot{\Omega}}$$

$$+ \frac{1}{4} \sin^2 \frac{i}{2} \left(1 - 10 \cos i - 15 \cos^2 i\right) \cos^2 \frac{i'}{2} \left(1 + 10 \cos i' - 15 \cos^2 i'\right) \frac{\cos (\lambda' + \omega - \Omega)}{\dot{\lambda}' + \dot{\omega} - \dot{\Omega}}$$

$$- \frac{5}{8} \sin i \left(1 - 2 \cos i - 3 \cos^2 i\right) \sin i' \left(1 + 2 \cos i' - 3 \cos^2 i'\right) \frac{\cos (\lambda' - 3\Omega' + \omega + 2\Omega)}{\dot{\lambda}' - 3\dot{\Omega}' + \dot{\omega} + 2\dot{\Omega}}$$

$$- \frac{5}{8} \sin i \left(1 + 2 \cos i - 3 \cos^2 i\right) \sin i' \left(1 - 2 \cos i' - 3 \cos^2 i'\right) \frac{\cos (\lambda' + \Omega' + \omega - 2\Omega)}{\dot{\lambda}' + \dot{\Omega}' + \dot{\omega} - 2\dot{\Omega}}$$

$$+ \frac{15}{4} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\cos (\lambda' - 4\Omega' + \omega + 3\Omega)}{\dot{\lambda}' - 4\dot{\Omega}' + \dot{\omega} + 3\dot{\Omega}}$$

$$+ \frac{15}{4} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\cos (\lambda' + 2\Omega' + \omega - 3\Omega)}{\dot{\lambda}' + 2\dot{\Omega}' + \dot{\omega} - 3\dot{\Omega}}$$

$$- \frac{3}{4} \sin i \left(1 - 5 \cos^2 i\right) \sin i' \left(1 - 5 \cos^2 i'\right) \frac{\cos (\lambda' - \Omega' - \omega)}{\dot{\lambda}' - \dot{\Omega}' - \dot{\omega}}$$

$$- \frac{1}{4} \cos^2 \frac{i}{2} \left(1 + 10 \cos i - 15 \cos^2 i\right) \cos^2 \frac{i'}{2} \left(1 + 10 \cos i' - 15 \cos^2 i'\right) \frac{\cos (\lambda' - \omega - \Omega)}{\dot{\lambda}' - \dot{\omega} - \dot{\Omega}}$$

$$- \frac{1}{4} \sin^2 \frac{i}{2} \left(1 - 10 \cos i - 15 \cos^2 i\right) \sin^2 \frac{i'}{2} \left(1 - 10 \cos i - 15 \cos^2 i\right) \frac{\cos(\lambda' - 2\Omega' - \omega + \Omega)}{\dot{\lambda}' - 2\dot{\Omega}' - \dot{\omega} + \dot{\Omega}}$$

$$- \frac{5}{8} \sin i \left(1 - 2 \cos i - 3 \cos^2 i\right) \sin i' \left(1 - 2 \cos i' - 3 \cos^2 i'\right) \frac{\cos(\lambda' + \Omega' - \omega - 2\Omega)}{\dot{\lambda}' + \dot{\Omega}' - \dot{\omega} - 2\dot{\Omega}}$$

$$- \frac{5}{8} \sin i \left(1 + 2 \cos i - 3 \cos^2 i\right) \sin i' \left(1 + 2 \cos i' - 3 \cos^2 i'\right) \frac{\cos(\lambda' - 3\Omega' - \omega + 2\Omega)}{\dot{\lambda}' - 3\dot{\Omega}' - \dot{\omega} + 2\dot{\Omega}}$$

$$- \frac{15}{4} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\cos(\lambda' + 2\Omega' - \omega - 3\Omega)}{\dot{\lambda}' + 2\dot{\Omega}' - \dot{\omega} - 3\dot{\Omega}}$$

$$- \frac{15}{4} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\cos(\lambda' - 4\Omega' - \omega + 3\Omega)}{\dot{\lambda}' - 4\dot{\Omega}' - \dot{\omega} + 3\dot{\Omega}}$$

$$+ \frac{5}{4} \sin i \left(1 - 5 \cos^2 i\right) \sin^3 i' \frac{\cos(3\lambda' - 3\Omega' + \omega)}{3\dot{\lambda}' - 3\dot{\Omega}' + \dot{\omega}}$$

$$- \frac{5}{4} \cos^2 \frac{i}{2} \left(1 + 10 \cos i - 15 \cos^2 i\right) \sin^2 i' \sin^2 \frac{i'}{2} \frac{\cos(3\lambda' - 4\Omega' + \omega + \Omega)}{3\dot{\lambda}' - 4\dot{\Omega}' + \dot{\omega} + \dot{\Omega}}$$

$$- \frac{5}{4} \sin^2 \frac{i}{2} \left(1 - 10 \cos i - 15 \cos^2 i\right) \sin^2 i' \cos^2 \frac{i'}{2} \frac{\cos(3\lambda' - 2\Omega' + \omega - \Omega)}{3\dot{\lambda}' - 2\dot{\Omega}' + \dot{\omega} - \dot{\Omega}}$$

$$- \frac{5}{2} \sin i \left(1 - 2 \cos i - 3 \cos^2 i\right) \sin i' \sin^4 \frac{i'}{2} \frac{\cos(3\lambda' - 5\Omega' + \omega + 2\Omega)}{3\dot{\lambda}' - 5\dot{\Omega}' + \dot{\omega} + 2\dot{\Omega}}$$

$$- \frac{5}{2} \sin i \left(1 + 2 \cos i - 3 \cos^2 i\right) \sin i' \cos^4 \frac{i'}{2} \frac{\cos(3\lambda' - \Omega' + \omega - 2\Omega)}{3\dot{\lambda}' - \dot{\Omega}' + \dot{\omega} - 2\dot{\Omega}}$$

$$+ 5 \sin^2 i \cos^2 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\cos(3\lambda' - 6\Omega' + \omega + 3\Omega)}{3\dot{\lambda}' - 6\dot{\Omega}' + \dot{\omega} + 3\dot{\Omega}}$$

$$+ 5 \sin^2 i \sin^2 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\cos(3\lambda' + \omega - 3\Omega)}{3\dot{\lambda}' + \dot{\omega} - 3\dot{\Omega}}$$

$$+ \frac{5}{4} \sin i \left(1 - 5 \cos^2 i\right) \sin^3 i' \frac{\cos(3\lambda' - 3\Omega' - \omega)}{3\dot{\lambda}' - 3\dot{\Omega}' - \dot{\omega}}$$

$$+ \frac{5}{4} \cos^2 \frac{i}{2} \left(1 + 10 \cos i - 15 \cos^2 i\right) \sin^2 i' \cos^2 \frac{i'}{2} \frac{\cos(3\lambda' - 2\Omega' - \omega - \Omega)}{3\dot{\lambda}' - 2\dot{\Omega}' - \dot{\omega} - \dot{\Omega}}$$

$$+ \frac{5}{4} \sin^2 \frac{i}{2} \left(1 - 10 \cos i - 15 \cos^2 i\right) \sin^2 i' \sin^2 \frac{i'}{2} \frac{\cos(3\lambda' - 4\Omega' - \omega + \Omega)}{3\dot{\lambda}' - 4\dot{\Omega}' - \dot{\omega} + \dot{\Omega}}$$

$$- \frac{5}{2} \sin i \left(1 - 2\cos i - 3\cos^2 i\right) \sin i' \cos^4 \frac{i'}{2} \; \frac{\cos(3\lambda' - \Omega' - \omega - 2\Omega)}{3\dot{\lambda}' - \dot{\Omega}' - \dot{\omega} - 2\dot{\Omega}}$$

$$- \frac{5}{2} \sin i \left(1 + 2\cos i - 3\cos^2 i\right) \sin i' \sin^4 \frac{i'}{2} \; \frac{\cos(3\lambda' - 5\Omega' - \omega + 2\Omega)}{3\dot{\lambda}' - 5\dot{\Omega}' - \dot{\omega} + 2\dot{\Omega}}$$

$$- 5 \sin^2 i \cos^2 \frac{i}{2} \cos^6 \frac{i'}{2} \; \frac{\cos(3\lambda' - \omega - 3\Omega)}{3\dot{\lambda}' - \dot{\omega} - 3\dot{\Omega}}$$

$$\left.- 5 \sin^2 i \sin^2 \frac{i}{2} \sin^6 \frac{i'}{2} \; \frac{\cos(3\lambda' - 6\Omega' - \omega + 3\Omega)}{3\dot{\lambda}' - 6\dot{\Omega}' - \dot{\omega} + 3\dot{\Omega}}\right]$$

$$- \frac{175}{64} m' \frac{a^3}{a'^4} e^3 \left[\frac{9}{16} \sin^3 i \sin i' \left(1 - 5\cos^2 i'\right) \frac{\cos(\lambda' - \Omega' + 3\omega)}{\dot{\lambda}' - \dot{\Omega}' + 3\dot{\omega}}\right.$$

$$- \frac{9}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 \frac{i'}{2} \left(1 - 10\cos i' - 15\cos^2 i'\right) \frac{\cos(\lambda' - 2\Omega' + 3\omega + \Omega)}{\dot{\lambda}' - 2\dot{\Omega}' + 3\dot{\omega} + \dot{\Omega}}$$

$$- \frac{9}{16} \sin^2 i \sin^2 \frac{i}{2} \cos^2 \frac{i'}{2} \left(1 + 10\cos i' - 15\cos^2 i'\right) \frac{\cos(\lambda' + 3\omega - \Omega)}{\dot{\lambda}' + 3\dot{\omega} - \dot{\Omega}}$$

$$- \frac{9}{8} \sin i \cos^4 \frac{i}{2} \sin i' \left(1 + 2\cos i' - 3\cos^2 i'\right) \frac{\cos(\lambda' - 3\Omega' + 3\omega + 2\Omega)}{\dot{\lambda}' - 3\dot{\Omega}' + 3\dot{\omega} + 2\dot{\Omega}}$$

$$- \frac{9}{8} \sin i \sin^4 \frac{i}{2} \sin i' \left(1 - 2\cos i' - 3\cos^2 i'\right) \frac{\cos(\lambda' + \Omega' + 3\omega - 2\Omega)}{\dot{\lambda}' + \dot{\Omega}' + 3\dot{\omega} - 2\dot{\Omega}}$$

$$+ \frac{9}{4} \cos^6 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \; \frac{\cos(\lambda' - 4\Omega' + 3\omega + 3\Omega)}{\dot{\lambda}' - 4\dot{\Omega}' + 3\dot{\omega} + 3\dot{\Omega}}$$

$$+ \frac{9}{4} \sin^6 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \; \frac{\cos(\lambda' + 2\Omega' + 3\omega - 3\Omega)}{\dot{\lambda}' + 2\dot{\Omega}' + 3\dot{\omega} - 3\dot{\Omega}}$$

$$+ \frac{9}{16} \sin^3 i \sin i' \left(1 - 5\cos^2 i'\right) \frac{\cos(\lambda' - \Omega' - 3\omega)}{\dot{\lambda}' - \dot{\Omega}' - 3\dot{\omega}}$$

$$+ \frac{9}{16} \sin^2 i \cos^2 \frac{i}{2} \cos^2 \frac{i'}{2} \left(1 + 10\cos i' - 15\cos^2 i'\right) \frac{\cos(\lambda' - 3\omega - \Omega)}{\dot{\lambda}' - 3\dot{\omega} - \dot{\Omega}}$$

$$+ \frac{9}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 \frac{i'}{2} \left(1 - 10\cos i' - 15\cos^2 i'\right) \frac{\cos(\lambda' - 2\Omega' - 3\omega + \Omega)}{\dot{\lambda}' - 2\dot{\Omega}' - 3\dot{\omega} + \dot{\Omega}}$$

$$- \frac{9}{8} \sin i \cos^4 \frac{i}{2} \sin i' \left(1 - 2\cos i' - 3\cos^2 i'\right) \frac{\cos(\lambda' + \Omega' - 3\omega - 2\Omega)}{\dot{\lambda}' + \dot{\Omega}' - 3\dot{\omega} - 2\dot{\Omega}}$$

$$- \frac{9}{8} \sin i \sin^4 \frac{i}{2} \sin i' \left(1 + 2\cos i' - 3\cos^2 i'\right) \frac{\cos(\lambda' - 3\Omega' - 3\omega + 2\Omega)}{\dot{\lambda}' - 3\dot{\Omega}' - 3\dot{\omega} + 2\dot{\Omega}}$$

$$- \frac{9}{4} \cos^6 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\cos(\lambda' + 2\Omega' - 3\omega - 3\Omega)}{\dot{\lambda}' + 2\dot{\Omega}' - 3\dot{\omega} - 3\dot{\Omega}}$$

$$- \frac{9}{4} \sin^6 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\cos(\lambda' - 4\Omega' - 3\omega + 3\Omega)}{\dot{\lambda}' - 4\dot{\Omega}' - 3\dot{\omega} + 3\dot{\Omega}}$$

$$- \frac{15}{16} \sin^3 i \sin^3 i' \frac{\cos(3\lambda' - 3\Omega' + 3\omega)}{3\dot{\lambda}' - 3\dot{\Omega}' + 3\dot{\omega}}$$

$$+ \frac{45}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\cos(3\lambda' - 4\Omega' + 3\omega + \Omega)}{3\dot{\lambda}' - 4\dot{\Omega}' + 3\dot{\omega} + \dot{\Omega}}$$

$$+ \frac{45}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\cos(3\lambda' - 2\Omega' + 3\omega - \Omega)}{3\dot{\lambda}' - 2\dot{\Omega}' + 3\dot{\omega} - \dot{\Omega}}$$

$$- \frac{9}{2} \sin i \cos^4 \frac{i}{2} \sin i' \sin^4 \frac{i'}{2} \frac{\cos(3\lambda' - 5\Omega' + 3\omega + 2\Omega)}{3\dot{\lambda}' - 5\dot{\Omega}' + 3\dot{\omega} + 2\dot{\Omega}}$$

$$- \frac{9}{2} \sin i \sin^4 \frac{i}{2} \sin i' \cos^4 \frac{i'}{2} \frac{\cos(3\lambda' - \Omega' + 3\omega - 2\Omega)}{3\dot{\lambda}' - \dot{\Omega}' + 3\dot{\omega} - 2\dot{\Omega}}$$

$$+ 3 \cos^6 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\cos(3\lambda' - 6\Omega' + 3\omega + 3\Omega)}{3\dot{\lambda}' - 6\dot{\Omega}' + 3\dot{\omega} + 3\dot{\Omega}}$$

$$+ 3 \sin^6 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\cos(3\lambda' + 3\omega - 3\Omega)}{3\dot{\lambda}' + 3\dot{\omega} - 3\dot{\Omega}}$$

$$- \frac{15}{16} \sin^3 i \sin^3 i' \frac{\cos(3\lambda' - 3\Omega' - 3\omega)}{3\dot{\lambda}' - 3\dot{\Omega}' - 3\dot{\omega}}$$

$$- \frac{45}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\cos(3\lambda' - 2\Omega' - 3\omega - \Omega)}{3\dot{\lambda}' - 2\dot{\Omega}' - 3\dot{\omega} - \dot{\Omega}}$$

$$- \frac{45}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\cos(3\lambda' - 4\Omega' - 3\omega + \Omega)}{3\dot{\lambda}' - 4\dot{\Omega}' - 3\dot{\omega} + \dot{\Omega}}$$

$$- \frac{9}{2} \sin i \cos^4 \frac{i}{2} \sin i' \cos^4 \frac{i'}{2} \frac{\cos(3\lambda' - \Omega' - 3\omega - 2\Omega)}{3\dot{\lambda}' - \dot{\Omega}' - 3\dot{\omega} - 2\dot{\Omega}}$$

$$- \frac{9}{2} \sin i \sin^4 \frac{i}{2} \sin i' \sin^4 \frac{i'}{2} \frac{\cos(3\lambda' - 5\Omega' - 3\omega + 2\Omega)}{3\dot{\lambda}' - 5\dot{\Omega}' - 3\dot{\omega} + 2\dot{\Omega}}$$

$$- 3 \cos^6 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\cos(3\lambda' - 3\omega - 3\Omega)}{3\dot{\lambda}' - 3\dot{\omega} - 3\dot{\Omega}}$$

$$- 3 \sin^6 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\cos(3\lambda' - 6\Omega' - 3\omega + 3\Omega)}{3\dot{\lambda}' - 6\dot{\Omega}' - 3\dot{\omega} + 3\dot{\Omega}} \Bigg] ;$$

$$A_2 = \int \frac{\partial R}{\partial \Omega}\, dt$$

$$= \frac{3}{32} n'^2 m' a^2 \left(1 + \frac{3}{2} e^2\right) \Bigg\{ - \sin 2i \sin 2i' \left[2\, \frac{\cos(\Omega' - \Omega)}{\dot\Omega' - \dot\Omega} + 3e' \frac{\cos(\lambda' - \omega' - \Omega)}{\dot\lambda' - \dot\omega' - \dot\Omega} - 3e' \frac{\cos(\lambda' - \omega' - 2\Omega' + \Omega)}{\dot\lambda' - \dot\omega' - 2\dot\Omega' + \dot\Omega} \right]$$

$$- 2 \sin^2 \sin i \sin^2 i' \left[\frac{2 \cos(2\Omega' - 2\Omega)}{2(\dot\Omega' - \dot\Omega)} + 3e' \frac{\cos(\lambda' - \omega' + \Omega' - 2\Omega)}{\dot\lambda' - \dot\omega' + \dot\Omega' - 2\dot\Omega} - 3e' \frac{\cos(\lambda' - \omega' - 3\Omega' + 2\Omega)}{\dot\lambda' - \dot\omega' - 3\dot\Omega' + 2\dot\Omega} \right]$$

$$- 4 \sin^2 i \cos^4 \frac{i'}{2} \left[2\, \frac{\cos(2\lambda' - 2\Omega)}{2(\dot\lambda' - \dot\Omega)} - e' \frac{\cos(\lambda' + \omega' + \Omega' - 2\Omega)}{\dot\lambda' + \dot\omega' + \dot\Omega' - 2\dot\Omega} + 7e' \frac{\cos(3\lambda' - \omega' - \Omega' - 2\Omega)}{3\dot\lambda' - \dot\omega' - \dot\Omega' - 2\dot\Omega} \right]$$

$$+ 2 \sin 2i \sin i' \cos^2 \frac{i'}{2} \left[2\, \frac{\cos(2\lambda' - \Omega' - \Omega)}{2\dot\lambda' - \dot\Omega' - \dot\Omega} - e' \frac{\cos(\lambda' + \omega' - \Omega)}{\dot\lambda' + \dot\omega' - \dot\Omega} + 7e' \frac{\cos(3\lambda' - \omega' - 2\Omega' - \Omega)}{3\dot\lambda' - \dot\omega' - 2\dot\Omega' - \dot\Omega} \right]$$

$$+ 4 \sin^2 i \sin^4 \frac{i'}{2} \left[\frac{2 \cos(2\lambda' - 4\Omega' + 2\Omega)}{2(\dot\lambda' - 2\dot\Omega' + \dot\Omega)} - e' \frac{\cos(\lambda' + \omega' - 3\Omega' + 2\Omega)}{\dot\lambda' + \dot\omega' - 3\dot\Omega' + 2\dot\Omega} + 7e' \frac{\cos(3\lambda' - \omega' - 5\Omega' + 2\Omega)}{3\dot\lambda' - \dot\omega' - 5\dot\Omega' + 2\dot\Omega} \right]$$

$$+ 2 \sin 2i \sin i' \sin^2 \frac{i'}{2} \left[2\, \frac{\cos(2\lambda' - 3\Omega' + \Omega)}{2\dot\lambda' - 3\dot\Omega' + \dot\Omega} - e' \frac{\cos(\lambda' + \omega' - 2\Omega' + \Omega)}{\dot\lambda' + \dot\omega' - 2\dot\Omega' + \dot\Omega} + 7e' \frac{\cos(3\lambda' - \omega' - 4\Omega' + \Omega)}{3\dot\lambda' - \dot\omega' - 4\dot\Omega' + \dot\Omega} \right] \Bigg\}$$

$$- \frac{15}{32} n'^2 m' a^2 e^2 \Bigg\{ 4 \cos^4 \frac{i}{2} \cos^4 \frac{i'}{2} \Bigg[2\, \frac{\cos(2\lambda' - 2\omega - 2\Omega)}{2(\dot\lambda' - \dot\omega - \dot\Omega)} - e' \frac{\cos(\lambda' + \omega' + \Omega' - 2\omega - 2\Omega)}{\dot\lambda' + \dot\omega' + \dot\Omega' - 2\dot\omega - 2\dot\Omega}$$

$$+ 7e' \frac{\cos(3\lambda' - \omega' - \Omega' - 2\omega - 2\Omega)}{3\dot\lambda' - \dot\omega' - \dot\Omega' - 2\dot\omega - 2\dot\Omega} \Bigg]$$

$$+ 2 \cos^4 \frac{i}{2} \sin^2 i' \left[\frac{2 \cos(2\Omega' - 2\omega - 2\Omega)}{2(\dot\Omega' - \dot\omega - \dot\Omega)} + 3e' \frac{\cos(\lambda' - \omega' + \Omega' - 2\omega - 2\Omega)}{\dot\lambda' - \dot\omega' + \dot\Omega' - 2\dot\omega - 2\dot\Omega} - 3e' \frac{\cos(\lambda' - \omega' - 3\Omega' + 2\omega + 2\Omega)}{\dot\lambda' - \dot\omega' - 3\dot\Omega' + 2\dot\omega + 2\dot\Omega} \right]$$

$$+ 4 \sin^4 \frac{i}{2} \cos^4 \frac{i'}{2} \left[2\, \frac{\cos(2\lambda' + 2\omega - 2\Omega)}{2(\dot\lambda' + \dot\omega - \dot\Omega)} - e' \frac{\cos(\lambda' + \omega' + \Omega' + 2\omega - 2\Omega)}{\dot\lambda' + \dot\omega' + \dot\Omega' + 2\dot\omega - 2\dot\Omega} + 7e' \frac{\cos(3\lambda' - \omega' - \Omega' + 2\omega - 2\Omega)}{3\dot\lambda' - \dot\omega' - \dot\Omega' + 2\dot\omega - 2\dot\Omega} \right]$$

$$- 4 \cos^4 \frac{i}{2} \sin^4 \frac{i'}{2} \left[2\, \frac{\cos(2\lambda' - 4\Omega' + 2\omega + 2\Omega)}{2(\dot\lambda' - 2\dot\Omega' + \dot\omega + \dot\Omega)} - e' \frac{\cos(\lambda' + \omega' - 3\Omega' + 2\omega + 2\Omega)}{\dot\lambda' + \dot\omega' - 3\dot\Omega' + 2\dot\omega + 2\dot\Omega} + 7e' \frac{\cos(3\lambda' - \omega' - 5\Omega' + 2\omega + 2\Omega)}{3\dot\lambda' - \dot\omega' - 5\dot\Omega' + 2\dot\omega + 2\dot\Omega} \right]$$

$$- 4 \sin^4 \frac{i}{2} \sin^4 \frac{i'}{2} \left[2\, \frac{\cos(2\lambda' - 4\Omega' - 2\omega + 2\Omega)}{2(\dot\lambda' - 2\dot\Omega' - \dot\omega + \dot\Omega)} - e' \frac{\cos(\lambda' + \omega' - 3\Omega' - 2\omega + 2\Omega)}{\dot\lambda' + \dot\omega' - 3\dot\Omega' - 2\dot\omega + 2\dot\Omega} + 7e' \frac{\cos(3\lambda' - \omega' - 5\Omega' - 2\omega + 2\Omega)}{3\dot\lambda' - \dot\omega' - 5\dot\Omega' - 2\dot\omega + 2\dot\Omega} \right]$$

$$+ 2 \sin i \cos^2 \frac{i}{2} \sin i' \cos^2 \frac{i'}{2} \left[2 \frac{\cos (2\lambda' - \Omega' - 2\omega - \Omega)}{2\dot{\lambda}' - \dot{\Omega}' - 2\dot{\omega} - \dot{\Omega}} - e' \frac{\cos (\lambda' + \omega' - 2\omega - \Omega)}{\dot{\lambda}' + \dot{\omega}' - 2\dot{\omega} - \dot{\Omega}} + 7 e' \frac{\cos (3\lambda' - \omega' - 2\Omega' - 2\omega - \Omega)}{3\dot{\lambda}' - \dot{\omega}' - 2\dot{\Omega}' - 2\dot{\omega} - \dot{\Omega}} \right]$$

$$- \sin i \cos^2 \frac{i}{2} \sin 2i' \left[2 \frac{\cos (\Omega' - 2\omega - \Omega)}{\dot{\Omega}' - 2\dot{\omega} - \dot{\Omega}} + 3e' \frac{\cos (\lambda' - \omega' - 2\omega - \Omega)}{\dot{\lambda}' - \dot{\omega}' - 2\dot{\omega} - \dot{\Omega}} - 3e' \frac{\cos (\lambda' - \omega' - 2\Omega' + 2\omega + \Omega)}{\dot{\lambda}' - \dot{\omega}' - 2\dot{\Omega}' + 2\dot{\omega} + \dot{\Omega}} \right]$$

$$+ 2 \sin^4 \frac{i}{2} \sin^2 i' \left[2 \frac{\cos (2\Omega' + 2\omega - 2\Omega)}{2(\dot{\Omega}' + \dot{\omega} - \dot{\Omega})} - 3e' \frac{\cos (\lambda' - \omega' - 3\Omega' - 2\omega + 2\Omega)}{\dot{\lambda}' - \dot{\omega}' - 3\dot{\Omega}' - 2\dot{\omega} + 2\dot{\Omega}} + 3e' \frac{\cos (\lambda' - \omega' + \Omega' + 2\omega - 2\Omega)}{\dot{\lambda}' - \dot{\omega}' + \dot{\Omega}' + 2\dot{\omega} - 2\dot{\Omega}} \right]$$

$$+ \sin i \sin^2 \frac{i}{2} \sin 2i' \left[2 \frac{\cos (\Omega' + 2\omega - \Omega)}{\dot{\Omega}' + 2\dot{\omega} - \dot{\Omega}} - 3e' \frac{\cos (\lambda' - \omega' - 2\Omega' - 2\omega + \Omega)}{\dot{\lambda}' - \dot{\omega}' - 2\dot{\Omega}' - 2\dot{\omega} + \dot{\Omega}} + 3e' \frac{\cos (\lambda' - \omega' + 2\omega - \Omega)}{\dot{\lambda}' - \dot{\omega}' + 2\dot{\omega} - \dot{\Omega}} \right]$$

$$- 2 \sin i \sin^2 \frac{i}{2} \sin i' \cos^2 \frac{i'}{2} \left[2 \frac{\cos (2\lambda' - \Omega' + 2\omega - \Omega)}{2\dot{\lambda}' - \dot{\Omega}' + 2\dot{\omega} - \dot{\Omega}} - e' \frac{\cos (\lambda' + \omega' + 2\omega - \Omega)}{\dot{\lambda}' + \dot{\omega}' + 2\dot{\omega} - \dot{\Omega}} + 7 e' \frac{\cos (3\lambda' - \omega' - 2\Omega' + 2\omega - \Omega)}{3\dot{\lambda}' - \dot{\omega}' - 2\dot{\Omega}' + 2\dot{\omega} - \dot{\Omega}} \right]$$

$$+ 2 \sin i \cos^2 \frac{i}{2} \sin i' \sin^2 \frac{i'}{2} \left[2 \frac{\cos (2\lambda' - 3\Omega' + 2\omega + \Omega)}{2\dot{\lambda}' - 3\dot{\Omega}' + 2\dot{\omega} + \dot{\Omega}} - e' \frac{\cos (\lambda' + \omega' - 2\Omega' + 2\omega + \Omega)}{\dot{\lambda}' + \dot{\omega}' - 2\dot{\Omega}' + 2\dot{\omega} + \dot{\Omega}} + 7 e' \frac{\cos (3\lambda' - \omega' - 4\Omega' + 2\omega + \Omega)}{3\dot{\lambda}' - \dot{\omega}' - 4\dot{\Omega}' + 2\dot{\omega} + \dot{\Omega}} \right]$$

$$- 2 \sin i \sin^2 \frac{i}{2} \sin i' \sin^2 \frac{i'}{2} \left[2 \frac{\cos (2\lambda' - 3\Omega' - 2\omega + \Omega)}{2\dot{\lambda}' - 3\dot{\Omega}' - 2\dot{\omega} + \dot{\Omega}} - e' \frac{\cos (\lambda' + \omega' - 2\Omega' - 2\omega + \Omega)}{\dot{\lambda}' + \dot{\omega}' - 2\dot{\Omega}' - 2\dot{\omega} + \dot{\Omega}} + 7 e' \frac{\cos (3\lambda' - \omega' - 4\Omega' - 2\omega + \Omega)}{3\dot{\lambda}' - \dot{\omega}' - 4\dot{\Omega}' - 2\dot{\omega} + \dot{\Omega}} \right] \Bigg\}$$

$$- m' \frac{a^3}{a'^4} \left(\frac{15}{64} e + \frac{45}{256} e^3 \right) \Bigg[\frac{1}{4} \cos^2 \frac{i}{2} \left(1 + 10 \cos i - 15 \cos^2 i \right) \sin^2 \frac{i'}{2} \left(1 - 10 \cos i' - 15 \cos^2 i' \right) \frac{\cos (\lambda' - 2\Omega' + \omega + \Omega)}{\dot{\lambda}' - 2\dot{\Omega}' + \dot{\omega} + \dot{\Omega}}$$

$$- \frac{1}{4} \sin^2 \frac{i}{2} \left(1 - 10 \cos i - 15 \cos^2 i \right) \cos^2 \frac{i'}{2} \left(1 + 10 \cos i' - 15 \cos^2 i' \right) \frac{\cos (\lambda' + \omega - \Omega)}{\dot{\lambda}' + \dot{\omega} - \dot{\Omega}}$$

$$- \frac{5}{4} \sin i \left(1 - 2 \cos i - 3 \cos^2 i \right) \sin i' \left(1 + 2 \cos i' - 3 \cos^2 i' \right) \frac{\cos (\lambda' - 3\Omega' + \omega + 2\Omega)}{\dot{\lambda}' - 3\dot{\Omega}' + \dot{\omega} + 2\dot{\Omega}}$$

$$+ \frac{5}{4} \sin i \left(1 + 2 \cos i - 3 \cos^2 i \right) \sin i' \left(1 - 2 \cos i' - 3 \cos^2 i' \right) \frac{\cos (\lambda' + \Omega' + \omega - 2\Omega)}{\dot{\lambda}' + \dot{\Omega}' + \dot{\omega} - 2\dot{\Omega}}$$

$$+ \frac{45}{4} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\cos (\lambda' - 4\Omega' + \omega + 3\Omega)}{\dot{\lambda}' - 4\dot{\Omega}' + \dot{\omega} + 3\dot{\Omega}}$$

$$- \frac{45}{4} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\cos (\lambda' + 2\Omega' + \omega - 3\Omega)}{\dot{\lambda}' + 2\dot{\Omega}' + \dot{\omega} - 3\dot{\Omega}}$$

$$- \frac{1}{4} \cos^2 \frac{i}{2} \left(1 + 10 \cos i - 15 \cos^2 i \right) \cos^2 \frac{i'}{2} \left(1 + 10 \cos i' - 15 \cos^2 i' \right) \frac{\cos (\lambda' - \omega - \Omega)}{\dot{\lambda}' - \dot{\omega} - \dot{\Omega}}$$

$$+\frac{1}{4}\sin^2\frac{i}{2}\left(1-10\cos i-15\cos^2 i\right)\sin^2\frac{i'}{2}\left(1-10\cos i'-15\cos^2 i'\right)\frac{\cos(\lambda'-2\Omega'-\omega+\Omega)}{\dot\lambda'-2\dot\Omega'-\dot\omega+\dot\Omega}$$

$$-\frac{5}{4}\sin i\left(1-2\cos i-3\cos^2 i\right)\sin i'\left(1-2\cos i'-3\cos^2 i'\right)\frac{\cos(\lambda'+\Omega'-\omega-2\Omega)}{\dot\lambda'+\dot\Omega'-\dot\omega-2\dot\Omega}$$

$$+\frac{5}{4}\sin i\left(1+2\cos i-3\cos^2 i\right)\sin i'\left(1+2\cos i'-3\cos^2 i'\right)\frac{\cos(\lambda'-3\Omega'-\omega+2\Omega)}{\dot\lambda'-3\dot\Omega'-\dot\omega+2\dot\Omega}$$

$$-\frac{45}{4}\sin^2 i\cos^2\frac{i}{2}\sin^2 i'\cos^2\frac{i'}{2}\frac{\cos(\lambda'+2\Omega'-\omega-3\Omega)}{\dot\lambda'+2\dot\Omega'-\dot\omega-3\dot\Omega}$$

$$+\frac{45}{4}\sin^2 i\sin^2\frac{i}{2}\sin^2 i'\sin^2\frac{i'}{2}\frac{\cos(\lambda'-4\Omega'-\omega+3\Omega)}{\dot\lambda'-4\dot\Omega'-\dot\omega+3\dot\Omega}$$

$$-\frac{5}{4}\cos^2\frac{i}{2}\left(1+10\cos i-15\cos^2 i\right)\sin^2 i'\sin^2\frac{i'}{2}\frac{\cos(3\lambda'-4\Omega'+\omega+\Omega)}{3\dot\lambda'-4\dot\Omega'+\dot\omega+\dot\Omega}$$

$$+\frac{5}{4}\sin^2\frac{i}{2}\left(1-10\cos i-15\cos^2 i\right)\sin^2 i'\cos^2\frac{i'}{2}\frac{\cos(3\lambda'-2\Omega'+\omega-\Omega)}{3\dot\lambda'-2\dot\Omega'+\dot\omega-\dot\Omega}$$

$$-5\sin i\left(1-2\cos i-3\cos^2 i\right)\sin i'\sin^4\frac{i'}{2}\frac{\cos(3\lambda'-5\Omega'+\omega+2\Omega)}{3\dot\lambda'-5\dot\Omega'+\dot\omega+2\dot\Omega}$$

$$+5\sin i\left(1+2\cos i-3\cos^2 i\right)\sin i'\cos^4\frac{i'}{2}\frac{\cos(3\lambda'-\Omega'+\omega-2\Omega)}{3\dot\lambda'-\dot\Omega'+\dot\omega-2\dot\Omega}$$

$$+15\sin^2 i\cos^2\frac{i}{2}\sin^6\frac{i'}{2}\frac{\cos(3\lambda'-6\Omega'+\omega+3\Omega)}{3\dot\lambda'-6\dot\Omega'+\dot\omega+3\dot\Omega}$$

$$-15\sin^2 i\sin^2\frac{i}{2}\cos^6\frac{i'}{2}\frac{\cos(3\lambda'+\omega-3\Omega)}{3\dot\lambda'+\dot\omega-3\dot\Omega}$$

$$+\frac{5}{4}\cos^2\frac{i}{2}\left(1+10\cos i-15\cos^2 i\right)\sin^2 i'\cos^2\frac{i'}{2}\frac{\cos(3\lambda'-2\Omega'-\omega-\Omega)}{3\dot\lambda'-2\dot\Omega'-\dot\omega-\dot\Omega}$$

$$-\frac{5}{4}\sin^2\frac{i}{2}\left(1-10\cos i-15\cos^2 i\right)\sin^2 i'\sin^2\frac{i'}{2}\frac{\cos(3\lambda'-4\Omega'-\omega+\Omega)}{3\dot\lambda'-4\dot\Omega'-\dot\omega+\dot\Omega}$$

$$- 5 \sin i \left(1 - 2\cos i - 3\cos^2 i\right) \sin i' \cos^4 \frac{i'}{2} \frac{\cos(3\lambda' - \Omega' - \omega - 2\Omega)}{3\dot{\lambda}' - \dot{\Omega}' - \dot{\omega} - 2\dot{\Omega}}$$

$$+ 5 \sin i \left(1 + 2\cos i - 3\cos^2 i\right) \sin i' \sin^4 \frac{i'}{2} \frac{\cos(3\lambda' - 5\Omega' - \omega + 2\Omega)}{3\dot{\lambda}' - 5\dot{\Omega}' - \dot{\omega} + 2\dot{\Omega}}$$

$$- 15 \sin^2 i \cos^2 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\cos(3\lambda' - \omega - 3\Omega)}{3\dot{\lambda}' - \dot{\omega} - 3\dot{\Omega}}$$

$$+ 15 \sin^2 i \sin^2 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\cos(3\lambda' - 6\Omega' - \omega + 3\Omega)}{3\dot{\lambda}' - 6\dot{\Omega}' - \dot{\omega} + 3\dot{\Omega}} \Bigg]$$

$$- \frac{175}{64} m' \frac{a^3}{a'^4} e^3 \Bigg[- \frac{3}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 \frac{i'}{2} \left(1 - 10\cos i' - 15\cos^2 i'\right) \frac{\cos(\lambda' - 2\Omega' + 3\omega + \Omega)}{\dot{\lambda}' - 2\dot{\Omega}' + 3\dot{\omega} + \dot{\Omega}}$$

$$+ \frac{3}{16} \sin^2 i \sin^2 \frac{i}{2} \cos^2 \frac{i'}{2} \left(1 + 10\cos i' - 15\cos^2 i'\right) \frac{\cos(\lambda' + 3\omega - \Omega)}{\dot{\lambda}' + 3\dot{\omega} - \dot{\Omega}}$$

$$- \frac{3}{4} \sin i \cos^4 \frac{i}{2} \sin i' \left(1 + 2\cos i' - 3\cos^2 i'\right) \frac{\cos(\lambda' - 3\Omega' + 3\omega + 2\Omega)}{\dot{\lambda}' - 3\dot{\Omega}' + 3\dot{\omega} + 2\dot{\Omega}}$$

$$+ \frac{3}{4} \sin i \sin^4 \frac{i}{2} \sin i' \left(1 - 2\cos i' - 3\cos^2 i'\right) \frac{\cos(\lambda' + \Omega' + 3\omega - 2\Omega)}{\dot{\lambda}' + \dot{\Omega}' + 3\dot{\omega} - 2\dot{\Omega}}$$

$$+ \frac{9}{4} \cos^6 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\cos(\lambda' - 4\Omega' + 3\omega + 3\Omega)}{\dot{\lambda}' - 4\dot{\Omega}' + 3\dot{\omega} + 3\dot{\Omega}}$$

$$- \frac{9}{4} \sin^6 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\cos(\lambda' + 2\Omega' + 3\omega - 3\Omega)}{\dot{\lambda}' + 2\dot{\Omega}' + 3\dot{\omega} - 3\dot{\Omega}}$$

$$+ \frac{3}{16} \sin^2 i \cos^2 \frac{i}{2} \cos^2 \frac{i'}{2} \left(1 + 10\cos i' - 15\cos^2 i'\right) \frac{\cos(\lambda' - 3\omega - \Omega)}{\dot{\lambda}' - 3\dot{\omega} - \dot{\Omega}}$$

$$- \frac{3}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 \frac{i'}{2} \left(1 - 10\cos i' - 15\cos^2 i'\right) \frac{\cos(\lambda' - 2\Omega' - 3\omega + \Omega)}{\dot{\lambda}' - 2\dot{\Omega}' - 3\dot{\omega} + \dot{\Omega}}$$

$$- \frac{3}{4} \sin i \cos^4 \frac{i}{2} \sin i' \left(1 - 2\cos i' - 3\cos^2 i'\right) \frac{\cos(\lambda' + \Omega' - 3\omega - 2\Omega)}{\dot{\lambda}' + \dot{\Omega}' - 3\dot{\omega} - 2\dot{\Omega}}$$

$$+ \frac{3}{4} \sin i \sin^4 \frac{i}{2} \sin i' \left(1 + 2\cos i' - 3\cos^2 i'\right) \frac{\cos(\lambda' - 3\Omega' - 3\omega + 2\Omega)}{\dot{\lambda}' - 3\dot{\Omega}' - 3\dot{\omega} + 2\dot{\Omega}}$$

$$- \frac{9}{4} \cos^6 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\cos(\lambda' + 2\Omega' - 3\omega - 3\Omega)}{\dot{\lambda}' + 2\dot{\Omega}' - 3\dot{\omega} - 3\dot{\Omega}}$$

$$+ \frac{9}{4} \sin^6 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\cos(\lambda' - 4\Omega' - 3\omega + 3\Omega)}{\dot{\lambda}' - 4\dot{\Omega}' - 3\dot{\omega} + 3\dot{\Omega}}$$

$$+ \frac{15}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\cos(3\lambda' - 4\Omega' + 3\omega + \Omega)}{3\dot{\lambda}' - 4\dot{\Omega}' + 3\dot{\omega} + \dot{\Omega}}$$

$$- \frac{15}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\cos(3\lambda' - 2\Omega' + 3\omega - \Omega)}{3\dot{\lambda}' - 2\dot{\Omega}' + 3\dot{\omega} - \dot{\Omega}}$$

$$- 3 \sin i \cos^4 \frac{i}{2} \sin i' \sin^4 \frac{i'}{2} \frac{\cos(3\lambda' - 5\Omega' + 3\omega + 2\Omega)}{3\dot{\lambda}' - 5\dot{\Omega}' + 3\dot{\omega} + 2\dot{\Omega}}$$

$$+ 3 \sin i \sin^4 \frac{i}{2} \sin i' \cos^4 \frac{i'}{2} \frac{\cos(3\lambda' - \Omega' + 3\omega - 2\Omega)}{3\dot{\lambda}' - \dot{\Omega}' + 3\dot{\omega} - 2\dot{\Omega}}$$

$$+ 3 \cos^6 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\cos(3\lambda' - 6\Omega' + 3\omega + 3\Omega)}{3\dot{\lambda}' - 6\dot{\Omega}' + 3\dot{\omega} + 3\dot{\Omega}}$$

$$- 3 \sin^6 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\cos(3\lambda' + 3\omega - 3\Omega)}{3\dot{\lambda}' + 3\dot{\omega} - 3\dot{\Omega}}$$

$$- \frac{15}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\cos(3\lambda' - 2\Omega' - 3\omega - \Omega)}{3\dot{\lambda}' - 2\dot{\Omega}' - 3\dot{\omega} - \dot{\Omega}}$$

$$+ \frac{15}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\cos(3\lambda' - 4\Omega' - 3\omega + \Omega)}{3\dot{\lambda}' - 4\dot{\Omega}' - 3\dot{\omega} + \dot{\Omega}}$$

$$- 3 \sin i \cos^4 \frac{i}{2} \sin i' \cos^4 \frac{i'}{2} \frac{\cos(3\lambda' - \Omega' - 3\omega - 2\Omega)}{3\dot{\lambda}' - \dot{\Omega}' - 3\dot{\omega} - 2\dot{\Omega}}$$

$$+ 3 \sin i \sin^4 \frac{i}{2} \sin i' \sin^4 \frac{i'}{2} \frac{\cos(3\lambda' - 5\Omega' - 3\omega + 2\Omega)}{3\dot{\lambda}' - 5\dot{\Omega}' - 3\dot{\omega} + 2\dot{\Omega}}$$

$$- 3 \cos^6 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\cos(3\lambda' - 3\omega - 3\Omega)}{3\dot{\lambda}' - 3\dot{\omega} - 3\dot{\Omega}}$$

$$\left. + 3 \sin^6 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\cos(3\lambda' - 6\Omega' - 3\omega + 3\Omega)}{3\dot{\lambda}' - 6\dot{\Omega}' - 3\dot{\omega} + 3\dot{\Omega}} \right] ;$$

$$A_3 = \int \frac{\partial R}{\partial a}\, dt$$

$$= 2n'^2 m' a \left\{ \left[\left(1 + \frac{3}{2} e^2\right) \int Pdt + \frac{15}{8} e^2 \int Qdt \right] - \frac{3a}{2a'} \left[\left(\frac{15}{16} e + \frac{45}{256} e^3\right) \int Sdt + \frac{175}{64} e^3 \int Vdt \right] \right\}$$

and

$$A_4 = \int \frac{\partial R}{\partial e}\, dt$$

$$= 3n'^2 m' a^2 \left\{ e \left[\int Pdt + \frac{5}{4} \int Qdt \right] - \frac{a}{a'} \left[\left(\frac{5}{64} + \frac{45}{256} e^2\right) \int Sdt + \frac{175}{64} e^2 \int Vdt \right] \right\}$$

where

$$\int Pdt = \frac{1}{4} \left(1 - \frac{3}{2} \sin^2 i\right) \left(1 - \frac{3}{2} \sin^2 i'\right) \left[t + 3e' \frac{\sin(\lambda' - \omega' - \Omega')}{\dot{\lambda}' - \dot{\omega}' - \dot{\Omega}'} \right]$$

$$+ \frac{3}{32} \sin 2i \sin 2i' \left[2 \frac{\sin(\Omega' - \Omega)}{\dot{\Omega}' - \dot{\Omega}} + 3e' \frac{\sin(\lambda' - \omega' - \Omega)}{\dot{\lambda}' - \dot{\omega}' - \dot{\Omega}} + 3e' \frac{\sin(\lambda' - \omega' - 2\Omega' + \Omega)}{\dot{\lambda}' - \dot{\omega}' - 2\dot{\Omega}' + \dot{\Omega}} \right]$$

$$+ \frac{3}{32} \sin^2 i \sin^2 i' \left[\frac{2\sin(2\Omega' - 2\Omega)}{2(\dot{\Omega}' - \dot{\Omega})} + 3e' \frac{\sin(\lambda' - \omega' + \Omega' - 2\Omega)}{\dot{\lambda}' - \dot{\omega}' + \dot{\Omega}' - 2\dot{\Omega}} + 3e' \frac{\sin(\lambda' - \omega' - 3\Omega' + 2\Omega)}{\dot{\lambda}' - \dot{\omega}' - 3\dot{\Omega}' + 2\dot{\Omega}} \right]$$

$$+ \frac{3}{16} \left(1 - \frac{3}{2} \sin^2 i\right) \sin^2 i' \left[\frac{2\sin(2\lambda' - 2\Omega')}{2(\dot{\lambda}' - \dot{\Omega}')} - e' \frac{\sin(\lambda' + \omega' - \Omega')}{\dot{\lambda}' + \dot{\omega}' - \dot{\Omega}'} + 7e' \frac{\sin(3\lambda' - \omega' - 3\Omega')}{3\dot{\lambda}' - \dot{\omega}' - 3\dot{\Omega}'} \right]$$

$$+ \frac{3}{16} \sin^2 i \cos^4 \frac{i'}{2} \left[\frac{2\sin(2\lambda' - 2\Omega)}{2(\dot{\lambda}' - \dot{\Omega})} - e' \frac{\sin(\lambda' + \omega' + \Omega' - 2\Omega)}{\dot{\lambda}' + \dot{\omega}' + \dot{\Omega}' - 2\dot{\Omega}} + 7e' \frac{\sin(3\lambda' - \omega' - \Omega' - 2\Omega)}{3\dot{\lambda}' - \dot{\omega}' - \dot{\Omega}' - 2\dot{\Omega}} \right]$$

$$- \frac{3}{16} \sin 2i \sin i' \cos^2 \frac{i'}{2} \left[2 \frac{\sin(2\lambda' - \Omega' - \Omega)}{2\dot{\lambda}' - \dot{\Omega}' - \dot{\Omega}} - e' \frac{\sin(\lambda' + \omega' - \Omega)}{\dot{\lambda}' + \dot{\omega}' - \dot{\Omega}} + 7e' \frac{\sin(3\lambda' - \omega' - 2\Omega' - \Omega)}{3\dot{\lambda}' - \dot{\omega}' - 2\dot{\Omega}' - \dot{\Omega}} \right]$$

$$+ \frac{3}{16} \sin^2 i \sin^4 \frac{i'}{2} \left[\frac{2\sin(2\lambda' - 4\Omega' + 2\Omega)}{2(\dot{\lambda}' - 2\dot{\Omega}' + \dot{\Omega})} - e' \frac{\sin(\lambda' + \omega' - 3\Omega' + 2\Omega)}{\dot{\lambda}' + \dot{\omega}' - 3\dot{\Omega}' + 2\dot{\Omega}} + 7e' \frac{\sin(3\lambda' - \omega' - 5\Omega' + 2\Omega)}{3\dot{\lambda}' - \dot{\omega}' - 5\dot{\Omega}' + 2\dot{\Omega}} \right]$$

$$+ \frac{3}{16} \sin 2i \sin i' \sin^2 \frac{i'}{2} \left[2 \frac{\sin(2\lambda' - 3\Omega' + \Omega)}{2\dot{\lambda}' - 3\dot{\Omega}' + \dot{\Omega}} - e' \frac{\sin(\lambda' + \omega' - 2\Omega' + \Omega)}{\dot{\lambda}' + \dot{\omega}' - 2\dot{\Omega}' + \dot{\Omega}} + 7e' \frac{\sin(3\lambda' - \omega' - 4\Omega' + \Omega)}{3\dot{\lambda}' - \dot{\omega}' - 4\dot{\Omega}' + \dot{\Omega}} \right],$$

$$
\begin{aligned}
\int Q\,dt = {} & \frac{1}{2}\cos^4\frac{i}{2}\cos^4\frac{i'}{2}\left[\frac{2\sin(2\lambda'-2\omega-2\Omega)}{2(\dot\lambda'-\dot\omega-\dot\Omega)} - e'\,\frac{\sin(\lambda'+\omega'+\Omega'-2\omega-2\Omega)}{\dot\lambda'+\dot\omega'+\dot\Omega'-2\dot\omega-2\dot\Omega} + 7e'\,\frac{\sin(3\lambda'-\omega'-\Omega'-2\omega-2\Omega)}{3\dot\lambda'-\dot\omega'-\dot\Omega'-2\dot\omega-2\dot\Omega}\right] \\
& + \frac{1}{4}\sin^2 i\left(1-\frac{3}{2}\sin^2 i'\right)\left[\frac{2\sin 2\omega}{2\dot\omega} + 3e'\,\frac{\sin(\lambda'-\omega'-\Omega'-2\omega)}{\dot\lambda'-\dot\omega'-\dot\Omega'-2\dot\omega} + 3e'\,\frac{\sin(\lambda'-\omega'-\Omega'+2\omega)}{\dot\lambda'-\dot\omega'-\dot\Omega'+2\dot\omega}\right] \\
& + \frac{1}{4}\cos^4\frac{i}{2}\sin^2 i'\left[\frac{2\sin(2\Omega'-2\omega-2\Omega)}{2(\dot\Omega'-\dot\omega-\dot\Omega)} + 3e'\,\frac{\sin(\lambda'-\omega'+\Omega'-2\omega-2\Omega)}{\dot\lambda'-\dot\omega'+\dot\Omega'-2\dot\omega-2\dot\Omega} + 3e'\,\frac{\sin(\lambda'-\omega'-3\Omega'+2\omega+2\Omega)}{\dot\lambda'-\dot\omega'-3\dot\Omega'+2\dot\omega+2\dot\Omega}\right] \\
& + \frac{1}{2}\sin^4\frac{i}{2}\cos^4\frac{i'}{2}\left[\frac{2\sin(2\lambda'+2\omega-2\Omega)}{2(\dot\lambda'+\dot\omega-\dot\Omega)} - e'\,\frac{\sin(\lambda'+\omega'+\Omega'+2\omega-2\Omega)}{\dot\lambda'+\dot\omega'+\dot\Omega'+2\dot\omega-2\dot\Omega} + 7e'\,\frac{\sin(3\lambda'-\omega'-\Omega'+2\omega-2\Omega)}{3\dot\lambda'-\dot\omega'-\dot\Omega'+2\dot\omega-2\dot\Omega}\right] \\
& + \frac{1}{2}\cos^4\frac{i}{2}\sin^4\frac{i'}{2}\left[\frac{2\sin(2\lambda'-4\Omega'+2\omega+2\Omega)}{2(\dot\lambda'-2\dot\Omega'+\dot\omega+\dot\Omega)} - e'\,\frac{\sin(\lambda'+\omega'-3\Omega'+2\omega+2\Omega)}{\dot\lambda'+\dot\omega'-3\dot\Omega'+2\dot\omega+2\dot\Omega} + 7e'\,\frac{\sin(3\lambda'-\omega'-5\Omega'+2\omega+2\Omega)}{3\dot\lambda'-\dot\omega'-5\dot\Omega'+2\dot\omega+2\dot\Omega}\right] \\
& + \frac{1}{2}\sin^4\frac{i}{2}\sin^4\frac{i'}{2}\left[\frac{2\sin(2\lambda'-4\Omega'-2\omega+2\Omega)}{2(\dot\lambda'-2\dot\Omega'-\dot\omega+\dot\Omega)} - e'\,\frac{\sin(\lambda'+\omega'-3\Omega'-2\omega+2\Omega)}{\dot\lambda'+\dot\omega'-3\dot\Omega'-2\dot\omega+2\dot\Omega} + 7e'\,\frac{\sin(3\lambda'-\omega'-5\Omega'-2\omega+2\Omega)}{3\dot\lambda'-\dot\omega'-5\dot\Omega'-2\dot\omega+2\dot\Omega}\right] \\
& + \frac{3}{16}\sin^2 i\sin^2 i'\left[\frac{2\sin(2\lambda'-2\Omega'-2\omega)}{2(\dot\lambda'-\dot\Omega'-\dot\omega)} - e'\,\frac{\sin(\lambda'+\omega'-\Omega'-2\omega)}{\dot\lambda'+\dot\omega'-\dot\Omega'-2\dot\omega} + 7e'\,\frac{\sin(3\lambda'-\omega'-3\Omega'-2\omega)}{3\dot\lambda'-\dot\omega'-3\dot\Omega'-2\dot\omega}\right] \\
& + \frac{3}{16}\sin^2 i\sin^2 i'\left[\frac{2\sin(2\lambda'-2\Omega'+2\omega)}{2(\dot\lambda'-\dot\Omega'+\dot\omega)} - e'\,\frac{\sin(\lambda'+\omega'-\Omega'+2\omega)}{\dot\lambda'+\dot\omega'-\dot\Omega'+2\dot\omega} + 7e'\,\frac{\sin(3\lambda'-\omega'-3\Omega'+2\omega)}{3\dot\lambda'-\dot\omega'-3\dot\Omega'+2\dot\omega}\right] \\
& + \frac{1}{2}\sin i\cos^2\frac{i}{2}\sin i'\cos^2\frac{i'}{2}\left[2\,\frac{\sin(2\lambda'-\Omega'-2\omega-\Omega)}{2\dot\lambda'-\dot\Omega'-2\dot\omega-\dot\Omega} - e'\,\frac{\sin(\lambda'+\omega'-2\omega-\Omega)}{\dot\lambda'+\dot\omega'-2\dot\omega-\dot\Omega} + 7e'\,\frac{\sin(3\lambda'-\omega'-2\Omega'-2\omega-\Omega)}{3\dot\lambda'-\dot\omega'-2\dot\Omega'-2\dot\omega-\dot\Omega}\right] \\
& - \frac{1}{4}\sin i\cos^2\frac{i}{2}\sin 2i'\left[2\,\frac{\sin(\Omega'-2\omega-\Omega)}{\dot\Omega'-2\dot\omega-\dot\Omega} + 3e'\,\frac{\sin(\lambda'-\omega'-2\omega-\Omega)}{\dot\lambda'-\dot\omega'-2\dot\omega-\dot\Omega} + 3e'\,\frac{\sin(\lambda'-\omega'-2\Omega'+2\omega+\Omega)}{\dot\lambda'-\dot\omega'-2\dot\Omega'+2\dot\omega+\dot\Omega}\right] \\
& + \frac{1}{4}\sin^4\frac{i}{2}\sin^2 i'\left[\frac{2\sin(2\Omega'+2\omega-2\Omega)}{2(\dot\Omega'+\dot\omega-\dot\Omega)} + 3e'\,\frac{\sin(\lambda'-\omega'-3\Omega'-2\omega+2\Omega)}{\dot\lambda'-\dot\omega'-3\dot\Omega'-2\dot\omega+2\dot\Omega} + 3e'\,\frac{\sin(\lambda'-\omega'+\Omega'+2\omega-2\Omega)}{\dot\lambda'-\dot\omega'+\dot\Omega'+2\dot\omega-2\dot\Omega}\right] \\
& + \frac{1}{4}\sin i\sin^2\frac{i}{2}\sin 2i'\left[2\,\frac{\sin(\Omega'+2\omega-\Omega)}{\dot\Omega'+2\dot\omega-\dot\Omega} + 3e'\,\frac{\sin(\lambda'-\omega'-2\Omega'-2\omega+\Omega)}{\dot\lambda'-\dot\omega'-2\dot\Omega'-2\dot\omega+\dot\Omega} + 3e'\,\frac{\sin(\lambda'-\omega'+2\omega-\Omega)}{\dot\lambda'-\dot\omega'+2\dot\omega-\dot\Omega}\right] \\
& - \frac{1}{2}\sin i\sin^2\frac{i}{2}\sin i'\cos^2\frac{i'}{2}\left[2\,\frac{\sin(2\lambda'-\Omega'+2\omega-\Omega)}{2\dot\lambda'-\dot\Omega'+2\dot\omega-\dot\Omega} - e'\,\frac{\sin(\lambda'+\omega'+2\omega-\Omega)}{\dot\lambda'+\dot\omega'+2\dot\omega-\dot\Omega} + 7e'\,\frac{\sin(3\lambda'-\omega'-2\Omega'+2\omega-\Omega)}{3\dot\lambda'-\dot\omega'-2\dot\Omega'+2\dot\omega-\dot\Omega}\right] \\
& - \frac{1}{2}\sin i\cos^2\frac{i}{2}\sin i'\sin^2\frac{i'}{2}\left[2\,\frac{\sin(2\lambda'-3\Omega'+2\omega+\Omega)}{2\dot\lambda'-3\dot\Omega'+2\dot\omega+\dot\Omega} - e'\,\frac{\sin(\lambda'+\omega'-2\Omega'+2\omega+\Omega)}{\dot\lambda'+\dot\omega'-2\dot\Omega'+2\dot\omega+\dot\Omega} + 7e'\,\frac{\sin(3\lambda'-\omega'-4\Omega'+2\omega+\Omega)}{3\dot\lambda'-\dot\omega'-4\dot\Omega'+2\dot\omega+\dot\Omega}\right] \\
& + \frac{1}{2}\sin i\sin^2\frac{i}{2}\sin i'\sin^2\frac{i'}{2}\left[2\,\frac{\sin(2\lambda'-3\Omega'-2\omega+\Omega)}{2\dot\lambda'-3\dot\Omega'-2\dot\omega+\dot\Omega} - e'\,\frac{\sin(\lambda'+\omega'-2\Omega'-2\omega+\Omega)}{\dot\lambda'+\dot\omega'-2\dot\Omega'-2\dot\omega+\dot\Omega} + 7e'\,\frac{\sin(3\lambda'-\omega'-4\Omega'-2\omega+\Omega)}{3\dot\lambda'-\dot\omega'-4\dot\Omega'-2\dot\omega+\dot\Omega}\right]
\end{aligned}
$$

$$\int \mathrm{Sdt} = \Bigg[-\frac{3}{4} \sin i \left(1 - 5\cos^2 i\right) \sin i' \left(1 - 5\cos^2 i'\right) \frac{\sin(\lambda' - \Omega' + \omega)}{\dot\lambda' - \dot\Omega' + \dot\omega}$$

$$+ \frac{1}{4} \cos^2 \frac{i}{2} \left(1 + 10\cos i - 15\cos^2 i\right) \sin^2 \frac{i'}{2} \left(1 - 10\cos i' - 15\cos^2 i'\right) \frac{\sin(\lambda' - 2\Omega' + \omega + \Omega)}{\dot\lambda' - 2\dot\Omega' + \dot\omega + \dot\Omega}$$

$$+ \frac{1}{4} \sin^2 \frac{i}{2} \left(1 - 10\cos i - 15\cos^2 i\right) \cos^2 \frac{i'}{2} \left(1 + 10\cos i' - 15\cos^2 i'\right) \frac{\sin(\lambda' + \omega - \Omega)}{\dot\lambda' + \dot\omega - \dot\Omega}$$

$$- \frac{5}{8} \sin i \left(1 - 2\cos i - 3\cos^2 i\right) \sin i' \left(1 + 2\cos i' - 3\cos^2 i'\right) \frac{\sin(\lambda' - 3\Omega' + \omega + 2\Omega)}{\dot\lambda' - 3\dot\Omega' + \dot\omega + 2\dot\Omega}$$

$$- \frac{5}{8} \sin i \left(1 + 2\cos i - 3\cos^2 i\right) \sin i' \left(1 - 2\cos i' - 3\cos^2 i'\right) \frac{\sin(\lambda' + \Omega' + \omega - 2\Omega)}{\dot\lambda' + \dot\Omega' + \dot\omega - 2\dot\Omega}$$

$$+ \frac{15}{4} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(\lambda' - 4\Omega' + \omega + 3\Omega)}{\dot\lambda' - 4\dot\Omega' + \dot\omega + 3\dot\Omega}$$

$$+ \frac{15}{4} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(\lambda' + 2\Omega' + \omega - 3\Omega)}{\dot\lambda' + 2\dot\Omega' + \dot\omega - 3\dot\Omega}$$

$$+ \frac{3}{4} \sin i \left(1 - 5\cos^2 i\right) \sin i' \left(1 - 5\cos^2 i'\right) \frac{\sin(\lambda' - \Omega' - \omega)}{\dot\lambda' - \dot\Omega' - \dot\omega}$$

$$+ \frac{1}{4} \cos^2 \frac{i}{2} \left(1 + 10\cos i - 15\cos^2 i\right) \cos^2 \frac{i'}{2} \left(1 + 10\cos i' - 15\cos^2 i'\right) \frac{\sin(\lambda' - \omega - \Omega)}{\dot\lambda' - \dot\omega - \dot\Omega}$$

$$+ \frac{1}{4} \sin^2 \frac{i}{2} \left(1 - 10\cos i - 15\cos^2 i\right) \sin^2 \frac{i'}{2} \left(1 - 10\cos i - 15\cos^2 i\right) \frac{\sin(\lambda' - 2\Omega' - \omega + \Omega)}{\dot\lambda' + \dot\Omega - \dot\omega - 2\dot\Omega}$$

$$+ \frac{5}{8} \sin i \left(1 - 2\cos i - 3\cos^2 i\right) \sin i' \left(1 - 2\cos i - 3\cos^2 i'\right) \frac{\sin(\lambda' + \Omega' - \omega - 2\Omega)}{\dot\lambda' + \dot\Omega' - \dot\omega - 2\dot\Omega}$$

$$+ \frac{5}{8} \sin i \left(1 + 2\cos i - 3\cos^2 i\right) \sin i' \left(1 + 2\cos i' - 3\cos^2 i'\right) \frac{\sin(\lambda' - 3\Omega' - \omega + 2\Omega)}{\dot\lambda' - 3\dot\Omega' - \dot\omega + 2\dot\Omega}$$

$$+ \frac{15}{4} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(\lambda' + 2\Omega' - \omega - 3\Omega)}{\dot\lambda' + 2\dot\Omega' - \dot\omega - 3\dot\Omega}$$

$$+ \frac{15}{4} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(\lambda' - 4\Omega' - \omega + 3\Omega)}{\dot\lambda' - 4\dot\Omega' - \dot\omega + 3\dot\Omega}$$

$$+ \frac{5}{4} \sin i \left(1 - 5\cos^2 i\right) \sin^3 i' \; \frac{\sin(3\lambda' - 3\Omega' + \omega)}{3\dot{\lambda}' - 3\dot{\Omega}' + \dot{\omega}}$$

$$- \frac{5}{4} \cos^2 \frac{i}{2} \left(1 + 10\cos i - 15\cos^2 i\right) \sin^2 i' \sin^2 \frac{i'}{2} \; \frac{\sin(3\lambda' - 4\Omega' + \omega + \Omega)}{3\dot{\lambda}' - 4\dot{\Omega}' + \dot{\omega} + \dot{\Omega}}$$

$$- \frac{5}{4} \sin^2 \frac{i}{2} \left(1 - 10\cos i - 15\cos^2 i\right) \sin^2 i' \cos^2 \frac{i'}{2} \; \frac{\sin(3\lambda' - 2\Omega' + \omega - \Omega)}{3\dot{\lambda}' - 2\dot{\Omega}' + \dot{\omega} - \dot{\Omega}}$$

$$- \frac{5}{2} \sin i \left(1 - 2\cos i - 3\cos^2 i\right) \sin i' \sin^4 \frac{i'}{2} \; \frac{\sin(3\lambda' - 5\Omega' + \omega + 2\Omega)}{3\dot{\lambda}' - 5\dot{\Omega}' + \dot{\omega} + 2\dot{\Omega}}$$

$$- \frac{5}{2} \sin i \left(1 + 2\cos i - 3\cos^2 i\right) \sin i' \cos^4 \frac{i'}{2} \; \frac{\sin(3\lambda' - \Omega' + \omega - 2\Omega)}{3\dot{\lambda}' - \dot{\Omega}' + \dot{\omega} - 2\dot{\Omega}}$$

$$+ 5 \sin^2 i \cos^2 \frac{i}{2} \sin^6 \frac{i'}{2} \; \frac{\sin(3\lambda' - 6\Omega' + \omega + 3\Omega)}{3\dot{\lambda}' - 6\dot{\Omega}' + \dot{\omega} + 3\dot{\Omega}}$$

$$+ 5 \sin^2 i \sin^2 \frac{i}{2} \cos^6 \frac{i'}{2} \; \frac{\sin(3\lambda' + \omega - 3\Omega)}{3\dot{\lambda}' + \dot{\omega} - 3\dot{\Omega}}$$

$$- \frac{5}{4} \sin i \left(1 - 5\cos^2 i\right) \sin^3 i' \; \frac{\sin(3\lambda' - 3\Omega' - \omega)}{3\dot{\lambda}' - 3\dot{\Omega}' - \dot{\omega}}$$

$$- \frac{5}{4} \cos^2 \frac{i}{2} \left(1 + 10\cos i - 15\cos^2 i\right) \sin^2 i' \cos^2 \frac{i'}{2} \; \frac{\sin(3\lambda' - 2\Omega' - \omega - \Omega)}{3\dot{\lambda}' - 2\dot{\Omega}' - \dot{\omega} - \dot{\Omega}}$$

$$- \frac{5}{4} \sin^2 \frac{i}{2} \left(1 - 10\cos i - 15\cos^2 i\right) \sin^2 i' \sin^2 \frac{i'}{2} \; \frac{\sin(3\lambda' - 4\Omega' - \omega + \Omega)}{3\dot{\lambda}' - 4\dot{\Omega}' - \dot{\omega} + \dot{\Omega}}$$

$$+ \frac{5}{2} \sin i \left(1 - 2\cos i - 3\cos^2 i\right) \sin i' \cos^4 \frac{i'}{2} \; \frac{\sin(3\lambda' - \Omega' - \omega - 2\Omega)}{3\dot{\lambda}' - \dot{\Omega}' - \dot{\omega} - 2\dot{\Omega}}$$

$$+ \frac{5}{2} \sin i \left(1 + 2\cos i \quad 3\cos^2 i\right) \sin i' \sin^4 \frac{i'}{2} \; \frac{\sin(3\lambda' - 5\Omega' - \omega + 2\Omega)}{3\dot{\lambda}' - 5\dot{\Omega}' - \dot{\omega} + 2\dot{\Omega}}$$

$$+ 5 \sin^2 i \cos^2 \frac{i}{2} \cos^6 \frac{i'}{2} \; \frac{\sin(3\lambda' - \omega - 3\Omega)}{3\dot{\lambda}' - \dot{\omega} - 3\dot{\Omega}}$$

$$\left. + 5 \sin^2 i \sin^2 \frac{i}{2} \sin^6 \frac{i'}{2} \; \frac{\sin(3\lambda' - 6\Omega' - \omega + 3\Omega)}{3\dot{\lambda}' - 6\dot{\Omega}' - \dot{\omega} + 3\dot{\Omega}} \right] ,$$

and

$$\int V dt = \Bigg[\frac{3}{16} \sin^3 i \sin i' \left(1 - 5\cos^2 i'\right) \frac{\sin(\lambda' - \Omega' + 3\omega)}{\dot{\lambda}' - \dot{\Omega}' + 3\dot{\omega}}$$

$$- \frac{3}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 \frac{i'}{2} \left(1 - 10\cos i' - 15\cos^2 i'\right) \frac{\sin(\lambda' - 2\Omega' + 3\omega + \Omega)}{\dot{\lambda}' - 2\dot{\Omega}' + 3\dot{\omega} + \dot{\Omega}}$$

$$- \frac{3}{16} \sin^2 i \sin^2 \frac{i}{2} \cos^2 \frac{i'}{2} \left(1 + 10\cos i' - 15\cos^2 i'\right) \frac{\sin(\lambda' + 3\omega - \Omega)}{\dot{\lambda}' + 3\dot{\omega} - \dot{\Omega}}$$

$$- \frac{3}{8} \sin i \cos^4 \frac{i}{2} \sin i' \left(1 + 2\cos i' - 3\cos^2 i'\right) \frac{\sin(\lambda' - 3\Omega' + 3\omega + 2\Omega)}{\dot{\lambda}' - 3\dot{\Omega}' + 3\dot{\omega} + 2\dot{\Omega}}$$

$$- \frac{3}{8} \sin i \sin^4 \frac{i}{2} \sin i' \left(1 - 2\cos i' - 3\cos^2 i'\right) \frac{\sin(\lambda' + \Omega' + 3\omega - 2\Omega)}{\dot{\lambda}' + \dot{\Omega}' + 3\dot{\omega} - 2\dot{\Omega}}$$

$$+ \frac{3}{4} \cos^6 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(\lambda' - 4\Omega' + 3\omega + 3\Omega)}{\dot{\lambda}' - 4\dot{\Omega}' + 3\dot{\omega} + 3\dot{\Omega}}$$

$$+ \frac{3}{4} \sin^6 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(\lambda' + 2\Omega' + 3\omega - 3\Omega)}{\dot{\lambda}' + 2\dot{\Omega}' + 3\dot{\omega} - 3\dot{\Omega}}$$

$$- \frac{3}{16} \sin^3 i \sin i' \left(1 - 5\cos^2 i'\right) \frac{\sin(\lambda' - \Omega' - 3\omega)}{\dot{\lambda}' - \dot{\Omega}' - 3\dot{\omega}}$$

$$- \frac{3}{16} \sin^2 i \cos^2 \frac{i}{2} \cos^2 \frac{i'}{2} \left(1 + 10\cos i' - 15\cos^2 i'\right) \frac{\sin(\lambda' - 3\omega - \Omega)}{\dot{\lambda}' - 3\dot{\omega} - \dot{\Omega}}$$

$$- \frac{3}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 \frac{i'}{2} \left(1 - 10\cos i' - 15\cos^2 i'\right) \frac{\sin(\lambda' - 2\Omega' - 3\omega + \Omega)}{\dot{\lambda}' - 2\dot{\Omega}' - 3\dot{\omega} + \dot{\Omega}}$$

$$+ \frac{3}{8} \sin i \cos^4 \frac{i}{2} \sin i' \left(1 - 2\cos i' - 3\cos^2 i'\right) \frac{\sin(\lambda' + \Omega' - 3\omega - 2\Omega)}{\dot{\lambda}' + \dot{\Omega}' - 3\dot{\omega} - 2\dot{\Omega}}$$

$$+ \frac{3}{8} \sin i \sin^4 \frac{i}{2} \sin i' \left(1 + 2\cos i' - 3\cos^2 i'\right) \frac{\sin(\lambda' - 3\Omega' - 3\omega + 2\Omega)}{\dot{\lambda}' - 3\dot{\Omega}' - 3\dot{\omega} + 2\dot{\Omega}}$$

$$+ \frac{3}{4} \cos^6 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(\lambda' + 2\Omega' - 3\omega - 3\Omega)}{\dot{\lambda}' + 2\dot{\Omega}' - 3\dot{\omega} - 3\dot{\Omega}}$$

$$+ \frac{3}{4} \sin^6 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(\lambda' - 4\Omega' - 3\omega + 3\Omega)}{\dot{\lambda}' - 4\dot{\Omega}' - 3\dot{\omega} + 3\dot{\Omega}}$$

$$- \frac{5}{16} \sin^3 i \sin^3 i' \frac{\sin (3\lambda' - 3\Omega' + 3\omega)}{3\dot{\lambda}' - 3\dot{\Omega}' + 3\dot{\omega}}$$

$$+ \frac{15}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin (3\lambda' - 4\Omega' + 3\omega + \Omega)}{3\dot{\lambda}' - 4\dot{\Omega}' + 3\dot{\omega} + \dot{\Omega}}$$

$$+ \frac{15}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin (3\lambda' - 2\Omega' + 3\omega - \Omega)}{3\dot{\lambda}' - 2\dot{\Omega}' + 3\dot{\omega} - \dot{\Omega}}$$

$$- \frac{3}{2} \sin i \cos^4 \frac{i}{2} \sin i' \sin^4 \frac{i'}{2} \frac{\sin (3\lambda' - 5\Omega' + 3\omega + 2\Omega)}{3\dot{\lambda}' - 5\dot{\Omega}' + 3\dot{\omega} + 2\dot{\Omega}}$$

$$- \frac{3}{2} \sin i \sin^4 \frac{i}{2} \sin i' \cos^4 \frac{i'}{2} \frac{\sin (3\lambda' - \Omega' + 3\omega - 2\Omega)}{3\dot{\lambda}' - \dot{\Omega}' + 3\dot{\omega} - 2\dot{\Omega}}$$

$$+ \cos^6 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\sin (3\lambda' - 6\Omega' + 3\omega + 3\Omega)}{3\dot{\lambda}' - 6\dot{\Omega}' + 3\dot{\omega} + 3\dot{\Omega}}$$

$$+ \sin^6 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\sin (3\lambda' + 3\omega - 3\Omega)}{3\dot{\lambda}' + 3\dot{\omega} - 3\dot{\Omega}}$$

$$+ \frac{15}{16} \sin^3 i \sin^3 i' \frac{\sin (3\lambda' - 3\Omega' - 3\omega)}{3\dot{\lambda}' - 3\dot{\Omega}' - 3\dot{\omega}}$$

$$+ \frac{15}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin (3\lambda' - 2\Omega' - 3\omega - \Omega)}{3\dot{\lambda}' - 2\dot{\Omega}' - 3\dot{\omega} - \dot{\Omega}}$$

$$+ \frac{15}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin (3\lambda' - 4\Omega' - 3\omega + \Omega)}{3\dot{\lambda}' - 4\dot{\Omega}' - 3\dot{\omega} + \dot{\Omega}}$$

$$+ \frac{3}{2} \sin i \cos^4 \frac{i}{2} \sin i' \cos^4 \frac{i'}{2} \frac{\sin (3\lambda' - \Omega' - 3\omega - 2\Omega)}{3\dot{\lambda}' - \dot{\Omega}' - 3\dot{\omega} - 2\dot{\Omega}}$$

$$+ \frac{3}{2} \sin i \sin^4 \frac{i}{2} \sin i' \sin^4 \frac{i'}{2} \frac{\sin (3\lambda' - 5\Omega' - 3\omega + 2\Omega)}{3\dot{\lambda}' - 5\dot{\Omega}' - 3\dot{\omega} + 2\dot{\Omega}}$$

$$+ \cos^6 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\sin (3\lambda' - 3\omega - 3\Omega)}{3\dot{\lambda}' - 3\dot{\omega} - 3\dot{\Omega}}$$

$$+ \sin^6 \frac{i}{2} \sin^6 \frac{i'}{2} \left. \frac{\sin (3\lambda' - 6\Omega' - 3\omega + 3\Omega)}{3\dot{\lambda}' - 6\dot{\Omega}' - 3\dot{\omega} + 3\dot{\Omega}} \right] ;$$

$$A_5 = \int \frac{\partial R}{\partial i}\, dt$$

$$= \frac{3}{16} n'^2 m' a^2 \left(1 + \frac{3}{2} e^2\right) \left\{ - 4 \sin i \cos i \left(1 - \frac{3}{2} \sin^2 i'\right) \left[t + 3e' \frac{\sin(\lambda' - \omega' - \Omega')}{\dot\lambda' - \dot\omega' - \dot\Omega'} \right] \right.$$

$$+ \cos 2i \sin 2i' \left[2 \frac{\sin(\Omega' - \Omega)}{\dot\Omega' - \dot\Omega} + 3e' \frac{\sin(\lambda' - \omega' - \Omega)}{\dot\lambda' - \dot\omega' - \dot\Omega} + 3e' \frac{\sin(\lambda' - \omega' - 2\Omega' + \Omega)}{\dot\lambda' - \dot\omega' - 2\dot\Omega' + \dot\Omega} \right]$$

$$+ \sin i \cos i \sin^2 i' \left[\frac{2 \sin 2(\Omega' - \Omega)}{2(\dot\Omega' - \dot\Omega)} + 3e' \frac{\sin(\lambda' - \omega' + \Omega' - 2\Omega)}{\dot\lambda' - \dot\omega' + \dot\Omega' - 2\dot\Omega} + 3e' \frac{\sin(\lambda' - \omega' - 3\Omega' + 2\Omega)}{\dot\lambda' - \dot\omega' - 3\dot\Omega' + 2\dot\Omega} \right]$$

$$- 3 \sin i \cos i \sin^2 i' \left[\frac{2 \sin 2(\lambda' - \Omega')}{2(\dot\lambda' - \dot\Omega')} - e' \frac{\sin(\lambda' + \omega' - \Omega')}{\dot\lambda' + \dot\omega' - \dot\Omega'} + 7e' \frac{\sin(3\lambda' - \omega' - 3\Omega')}{3\dot\lambda' - \dot\omega' - 3\dot\Omega'} \right]$$

$$+ 2 \sin i \cos i \cos^4 \frac{i'}{2} \left[\frac{2 \sin 2(\lambda' - \Omega)}{2(\dot\lambda' - \dot\Omega)} - e' \frac{\sin(\lambda' + \omega' + \Omega' - 2\Omega)}{\dot\lambda' + \dot\omega' + \dot\Omega' - 2\dot\Omega} + 7e' \frac{\sin(3\lambda' - \omega' - \Omega' - 2\Omega)}{3\dot\lambda' - \dot\omega' - \dot\Omega' - 2\dot\Omega} \right]$$

$$- 2 \cos 2i \sin i' \cos^2 \frac{i'}{2} \left[2 \frac{\sin(2\lambda' - \Omega' - \Omega)}{2\dot\lambda' - \dot\Omega' - \dot\Omega} - e' \frac{\sin(\lambda' + \omega' - \Omega)}{\dot\lambda' + \dot\omega' - \dot\Omega} + 7e' \frac{\sin(3\lambda' - \omega' - 2\Omega' - \Omega)}{3\dot\lambda' - \dot\omega' - 2\dot\Omega' - \dot\Omega} \right]$$

$$+ 2 \sin i \cos i \sin^4 \frac{i'}{2} \left[\frac{2 \sin 2(\lambda' - 2\Omega' + \Omega)}{2(\dot\lambda' - 2\dot\Omega' + \dot\Omega)} - e' \frac{\sin(\lambda' + \omega' - 3\Omega' + 2\Omega)}{\dot\lambda' + \dot\omega' - 3\dot\Omega' + 2\dot\Omega} + 7e' \frac{\sin(3\lambda' - \omega' - 5\Omega' + 2\Omega)}{3\dot\lambda' - \dot\omega' - 5\dot\Omega' + 2\dot\Omega} \right]$$

$$\left. + 2 \cos 2i \sin i' \sin^2 \frac{i'}{2} \left[2 \frac{\sin(2\lambda' - 3\Omega' + \Omega)}{2\dot\lambda' - 3\dot\Omega' + \dot\Omega} - e' \frac{\sin(\lambda' + \omega' - 2\Omega' + \Omega)}{\dot\lambda' + \dot\omega' - 2\dot\Omega' + \dot\Omega} + 7e' \frac{\sin(3\lambda' - \omega' - 4\Omega' + \Omega)}{3\dot\lambda' - \dot\omega' - 4\dot\Omega' + \dot\Omega} \right] \right\}$$

$$+ \frac{15}{64} n'^2 m' a^2 e^2 \left\{ - 8 \sin \frac{i}{2} \cos^3 \frac{i}{2} \cos^4 \frac{i'}{2} \left[\frac{2 \sin 2(\lambda' - \omega - \Omega)}{2(\dot\lambda' - \dot\omega - \dot\Omega)} - e' \frac{\sin(\lambda' + \omega' + \Omega' - 2\omega - 2\Omega)}{\dot\lambda' + \dot\omega' + \dot\Omega' - 2\dot\omega - 2\dot\Omega} \right. \right.$$

$$\left. + 7e' \frac{\sin(3\lambda' - \omega' - \Omega' - 2\omega - 2\Omega)}{3\dot\lambda' - \dot\omega' - \dot\Omega' - 2\dot\omega - 2\dot\Omega} \right]$$

$$+ 4 \sin i \cos i \left(1 - \frac{3}{2} \sin^2 i'\right) \left[\frac{2 \sin 2\omega}{2\dot\omega} + 3e' \frac{\sin(\lambda' - \omega' - \Omega' - 2\omega)}{\dot\lambda' - \dot\omega' - \dot\Omega' - 2\dot\omega} + 3e' \frac{\sin(\lambda' - \omega' - \Omega' + 2\omega)}{\dot\lambda' - \dot\omega' - \dot\Omega' + 2\dot\omega} \right]$$

$$- 4 \sin \frac{i}{2} \cos^3 \frac{i}{2} \sin^2 i' \left[\frac{2 \sin (2\Omega' - 2\omega - 2\Omega)}{2(\dot\Omega' - \dot\omega - \dot\Omega)} + 3e' \frac{\sin(\lambda' - \omega' + \Omega' - 2\omega - 2\Omega)}{\dot\lambda' - \dot\omega' + \dot\Omega' - 2\dot\omega - 2\dot\Omega} + 3e' \frac{\sin(\lambda' - \omega' - 3\Omega' + 2\omega + 2\Omega)}{\dot\lambda' - \dot\omega' - 3\dot\Omega' + 2\dot\omega + 2\dot\Omega} \right]$$

$$+ 8 \cos\frac{i}{2} \sin^3\frac{i}{2} \cos^4\frac{i'}{2} \left[\frac{2\sin(2\lambda' + 2\omega - 2\Omega)}{2(\dot\lambda' + \dot\omega - \dot\Omega)} - e' \frac{\sin(\lambda' + \omega' + \Omega' + 2\omega - 2\Omega)}{\dot\lambda' + \dot\omega' + \dot\Omega' + 2\dot\omega - 2\dot\Omega} + 7e' \frac{\sin(3\lambda' - \omega' - \Omega' + 2\omega - 2\Omega)}{3\dot\lambda' - \dot\omega' - \dot\Omega' + 2\dot\omega - 2\dot\Omega}\right]$$

$$- 8 \sin\frac{i}{2} \cos^3\frac{i}{2} \sin^4\frac{i'}{2} \left[\frac{2\sin(2\lambda' - 4\Omega' + 2\omega + 2\Omega)}{2(\dot\lambda' - 2\dot\Omega' + \dot\omega + \dot\Omega)} - e' \frac{\sin(\lambda' + \omega' - 3\Omega' + 2\omega + 2\Omega)}{\dot\lambda' + \dot\omega' - 3\dot\Omega' + 2\dot\omega + 2\dot\Omega} + 7e' \frac{\sin(3\lambda' - \omega' - 5\Omega' + 2\omega + 2\Omega)}{3\dot\lambda' - \dot\omega' - 5\dot\Omega' + 2\dot\omega + 2\dot\Omega}\right]$$

$$+ 8 \cos\frac{i}{2} \sin^3\frac{i}{2} \sin^4\frac{i'}{2} \left[\frac{2\sin(2\lambda' - 4\Omega' - 2\omega + 2\Omega)}{2(\dot\lambda' - 2\dot\Omega' - \dot\omega + \dot\Omega)} - e' \frac{\sin(\lambda' + \omega' - 3\Omega' - 2\omega + 2\Omega)}{\dot\lambda' + \dot\omega' - 3\dot\Omega' - 2\dot\omega + 2\dot\Omega} + 7e' \frac{\sin(3\lambda' - \omega' - 5\Omega' - 2\omega + 2\Omega)}{3\dot\lambda' - \dot\omega' - 5\dot\Omega' - 2\dot\omega + 2\dot\Omega}\right]$$

$$+ 3 \sin i \cos i \sin^2 i' \left[\frac{2\sin(2\lambda' - 2\Omega' - 2\omega)}{2(\dot\lambda' - \dot\Omega' - \dot\omega)} - e' \frac{\sin(\lambda' + \omega' - \Omega' - 2\omega)}{\dot\lambda' + \dot\omega' - \dot\Omega' - 2\dot\omega} + 7e' \frac{\sin(3\lambda' - \omega' - 3\Omega' - 2\omega)}{3\dot\lambda' - \dot\omega' - 3\dot\Omega' - 2\dot\omega}\right]$$

$$+ 3 \sin i \cos i \sin^2 i' \left[\frac{2\sin(2\lambda' - 2\Omega' + 2\omega)}{2(\dot\lambda' - \dot\Omega' + \dot\omega)} - e' \frac{\sin(\lambda' + \omega' - \Omega' + 2\omega)}{\dot\lambda' + \dot\omega' - \dot\Omega' + 2\dot\omega} + 7e' \frac{\sin(3\lambda' - \omega' - 3\Omega' + 2\omega)}{3\dot\lambda' - \dot\omega' - 3\dot\Omega' + 2\dot\omega}\right]$$

$$+ 2(\cos i + \cos 2i) \sin i' \cos^2\frac{i'}{2} \left[2 \frac{\sin(2\lambda' - \Omega' - 2\omega - \Omega)}{2\dot\lambda' - \dot\Omega' - 2\dot\omega - \dot\Omega} - e' \frac{\sin(\lambda' + \omega' - 2\omega - \Omega)}{\dot\lambda' + \dot\omega' - 2\dot\omega - \dot\Omega} + 7e' \frac{\sin(3\lambda' - \omega' - 2\Omega' - 2\omega - \Omega)}{3\dot\lambda' - \dot\omega' - 2\dot\Omega' - 2\dot\omega - \dot\Omega}\right]$$

$$- (\cos i + \cos 2i) \sin 2i' \left[2 \frac{\sin(\Omega' - 2\omega - \Omega)}{\dot\Omega' - 2\dot\omega - \dot\Omega} + 3e' \frac{\sin(\lambda' - \omega' - 2\omega - \Omega)}{\dot\lambda' - \dot\omega' - 2\dot\omega - \dot\Omega} + 3e' \frac{\sin(\lambda' - \omega' - 2\Omega' + 2\omega + \Omega)}{\dot\lambda' - \dot\omega' - 2\dot\Omega' + 2\dot\omega + \dot\Omega}\right]$$

$$+ 4 \cos\frac{i}{2} \sin^3\frac{i}{2} \sin^2 i' \left[\frac{2\sin(2\Omega' + 2\omega - 2\Omega)}{2(\dot\Omega' + \dot\omega - \dot\Omega)} + 3e' \frac{\sin(\lambda' - \omega' - 3\Omega' - 2\omega + 2\Omega)}{\dot\lambda' - \dot\omega' - 3\dot\Omega' - 2\dot\omega + 2\dot\Omega} + 3e' \frac{\sin(\lambda' - \omega' + \Omega' + 2\omega - 2\Omega)}{\dot\lambda' - \dot\omega' + \dot\Omega' + 2\dot\omega - 2\dot\Omega}\right]$$

$$+ (\cos i - \cos 2i) \sin 2i' \left[2 \frac{\sin(\Omega' + 2\omega - \Omega)}{\dot\Omega' + 2\dot\omega - \dot\Omega} + 3e' \frac{\sin(\lambda' - \omega' - 2\Omega' - 2\omega + \Omega)}{\dot\lambda' - \dot\omega' - 2\dot\Omega' - 2\dot\omega + \dot\Omega} + 3e' \frac{\sin(\lambda' - \omega' + 2\omega - \Omega)}{\dot\lambda' - \dot\omega' + 2\dot\omega - \dot\Omega}\right]$$

$$- 2(\cos i - \cos 2i) \sin i' \cos^2\frac{i'}{2} \left[2 \frac{\sin(2\lambda' - \Omega' + 2\omega - \Omega)}{2\dot\lambda' - \dot\Omega' + 2\dot\omega - \dot\Omega} - e' \frac{\sin(\lambda' + \omega' + 2\omega - \Omega)}{\dot\lambda' + \dot\omega' + 2\dot\omega - \dot\Omega} + 7e' \frac{\sin(3\lambda' - \omega' - 2\Omega' + 2\omega - \Omega)}{3\dot\lambda' - \dot\omega' - 2\dot\Omega' + 2\dot\omega - \dot\Omega}\right]$$

$$- 2(\cos i + \cos 2i) \sin i' \sin^2\frac{i'}{2} \left[2 \frac{\sin(2\lambda' - 3\Omega' + 2\omega + \Omega)}{2\dot\lambda' - 3\dot\Omega' + 2\dot\omega + \dot\Omega} - e' \frac{\sin(\lambda' + \omega' - 2\Omega' + 2\omega + \Omega)}{\dot\lambda' + \dot\omega' - 2\dot\Omega' + 2\dot\omega + \dot\Omega} + 7e' \frac{\sin(3\lambda' - \omega' - 4\Omega' + 2\omega + \Omega)}{3\dot\lambda' - \dot\omega' - 4\dot\Omega' + 2\dot\omega + \dot\Omega}\right]$$

$$+ 2(\cos i - \cos 2i) \sin i' \sin^2\frac{i'}{2} \left[2 \frac{\sin(2\lambda' - 3\Omega' - 2\omega + \Omega)}{2\dot\lambda' - 3\dot\Omega' - 2\dot\omega + \dot\Omega} - e' \frac{\sin(\lambda' + \omega' - 2\Omega' - 2\omega + \Omega)}{\dot\lambda' + \dot\omega' - 2\dot\Omega' - 2\dot\omega + \dot\Omega} + 7e' \frac{\sin(3\lambda' - \omega' - 4\Omega' - 2\omega + \Omega)}{3\dot\lambda' - \dot\omega' - 4\dot\Omega' - 2\dot\omega + \dot\Omega}\right]\Bigg\}$$

$$- m' \frac{a^3}{a'^4} \left(\frac{15}{64} e + \frac{45}{256} e^3\right)\Bigg[- \frac{3}{4} \cos i \left(11 - 15\cos^2 i\right) \sin i' \left(1 - 5\cos^2 i'\right) \frac{\sin(\lambda' - \Omega' + \omega)}{\dot\lambda' - \dot\Omega' + \dot\omega}$$

$$- \frac{1}{8} \sin i \left(11 - 10 \cos i - 45 \cos^2 i\right) \sin^2 \frac{i'}{2} \left(1 - 10 \cos i' - 15 \cos^2 i'\right) \frac{\sin(\lambda' - 2\Omega' + \omega + \Omega)}{\dot\lambda' - 2\dot\Omega' + \dot\omega + \dot\Omega}$$

$$+ \frac{1}{8} \sin i \left(11 + 10 \cos i - 45 \cos^2 i\right) \cos^2 \frac{i'}{2} \left(1 + 10 \cos i' - 15 \cos^2 i'\right) \frac{\sin(\lambda' + \omega - \Omega)}{\dot\lambda' + \dot\omega - \dot\Omega}$$

$$- \frac{5}{8} \left(2 + 7 \cos i - 4 \cos^2 i - 9 \cos^3 i\right) \sin i' \left(1 + 2 \cos i' - 3 \cos^2 i'\right) \frac{\sin(\lambda' - 3\Omega' + \omega + 2\Omega)}{\dot\lambda' - 3\dot\Omega' + \dot\omega + 2\dot\Omega}$$

$$+ \frac{5}{8} \left(2 - 7 \cos i - 4 \cos^2 i + 9 \cos^3 i\right) \sin i' \left(1 - 2 \cos i' - 3 \cos^2 i'\right) \frac{\sin(\lambda' + \Omega' + \omega - 2\Omega)}{\dot\lambda' + \dot\Omega' + \dot\omega - 2\dot\Omega}$$

$$- \frac{15}{8} \sin i \left(1 - 2 \cos i - 3 \cos^2 i\right) \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(\lambda' - 4\Omega' + \omega + 3\Omega)}{\dot\lambda' - 4\dot\Omega' + \dot\omega + 3\dot\Omega}$$

$$+ \frac{15}{8} \sin i \left(1 + 2 \cos i - 3 \cos^2 i\right) \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(\lambda' + 2\Omega' + \omega - 3\Omega)}{\dot\lambda' + 2\dot\Omega' + \dot\omega - 3\dot\Omega}$$

$$+ \frac{3}{4} \cos i \left(11 - 15 \cos^2 i\right) \sin i' \left(1 - 5 \cos^2 i'\right) \frac{\sin(\lambda' - \Omega' - \omega)}{\dot\lambda' - \dot\Omega' - \dot\omega}$$

$$- \frac{1}{8} \sin i \left(11 - 10 \cos i - 45 \cos^2 i\right) \cos^2 \frac{i'}{2} \left(1 + 10 \cos i' - 15 \cos^2 i'\right) \frac{\sin(\lambda' - \omega - \Omega)}{\dot\lambda' - \dot\omega - \dot\Omega}$$

$$+ \frac{1}{8} \sin i \left(11 + 10 \cos i - 45 \cos^2 i\right) \sin^2 \frac{i'}{2} \left(1 - 10 \cos i' - 15 \cos^2 i'\right) \frac{\sin(\lambda' - 2\Omega' - \omega + \Omega)}{\dot\lambda' - 2\dot\Omega' - \dot\omega + \dot\Omega}$$

$$+ \frac{5}{8} \left(2 + 7 \cos i - 4 \cos^2 i - 9 \cos^3 i\right) \sin i' \left(1 - 2 \cos i' - 3 \cos^2 i'\right) \frac{\sin(\lambda' + \Omega' - \omega - 2\Omega)}{\dot\lambda' + \dot\Omega' - \dot\omega - 2\dot\Omega}$$

$$- \frac{5}{8} \left(2 - 7 \cos i - 4 \cos^2 i + 9 \cos^3 i\right) \sin i' \left(1 + 2 \cos i' - 3 \cos^2 i'\right) \frac{\sin(\lambda' - 3\Omega' - \omega + 2\Omega)}{\dot\lambda' - 3\dot\Omega' - \dot\omega + 2\dot\Omega}$$

$$- \frac{15}{8} \sin i \left(1 - 2 \cos i - 3 \cos^2 i\right) \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(\lambda' + 2\Omega' - \omega - 3\Omega)}{\dot\lambda' + 2\dot\Omega' - \dot\omega - 3\dot\Omega}$$

$$+ \frac{15}{8} \sin i \left(1 + 2 \cos i - 3 \cos^2 i\right) \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(\lambda' - 4\Omega' - \omega + 3\Omega)}{\dot\lambda' - 4\dot\Omega' - \dot\omega + 3\dot\Omega}$$

$$+ \frac{5}{4} \cos i \left(11 - 15 \cos^2 i\right) \sin^3 i' \frac{\sin(3\lambda' - 3\Omega' + \omega)}{3\dot\lambda' - 3\dot\Omega' + \dot\omega}$$

$$+\frac{5}{8}\sin i\left(11-10\cos i-45\cos^2 i\right)\sin^2 i'\sin^2\frac{i'}{2}\;\frac{\sin(3\lambda'-4\Omega'+\omega+\Omega)}{3\dot{\lambda}'-4\dot{\Omega}'+\dot{\omega}+\dot{\Omega}}$$

$$-\frac{5}{8}\sin i\left(11+10\cos i-45\cos^2 i\right)\sin^2 i'\cos^2\frac{i'}{2}\;\frac{\sin(3\lambda'-2\Omega'+\omega-\Omega)}{3\dot{\lambda}'-2\dot{\Omega}'+\dot{\omega}-\dot{\Omega}}$$

$$-\frac{5}{2}\left(2+7\cos i-4\cos^2 i-9\cos^3 i\right)\sin i'\sin^4\frac{i'}{2}\;\frac{\sin(3\lambda'-5\Omega'+\omega+2\Omega)}{3\dot{\lambda}'-5\dot{\Omega}'+\dot{\omega}+2\dot{\Omega}}$$

$$+\frac{5}{2}\left(2-7\cos i-4\cos^2 i+9\cos^3 i\right)\sin i'\cos^4\frac{i'}{2}\;\frac{\sin(3\lambda'-\Omega'+\omega-2\Omega)}{3\dot{\lambda}'-\dot{\Omega}'+\dot{\omega}-2\dot{\Omega}}$$

$$-\frac{5}{2}\sin i\left(1-2\cos i-3\cos^2 i\right)\sin^6\frac{i'}{2}\;\frac{\sin(3\lambda'-6\Omega'+\omega+3\Omega)}{3\dot{\lambda}'-6\dot{\Omega}'+\dot{\omega}+3\dot{\Omega}}$$

$$+\frac{5}{2}\sin i\left(1+2\cos i-3\cos^2 i\right)\cos^6\frac{i'}{2}\;\frac{\sin(3\lambda'+\omega-3\Omega)}{3\dot{\lambda}+\dot{\omega}-3\dot{\Omega}}$$

$$-\frac{5}{4}\cos i\left(11-15\cos^2 i\right)\sin^3 i'\;\frac{\sin(3\lambda'-3\Omega'-\omega)}{3\dot{\lambda}-3\dot{\Omega}'-\dot{\omega}}$$

$$+\frac{5}{8}\sin i\left(11-10\cos i-45\cos^2 i\right)\sin^2 i'\cos^2\frac{i'}{2}\;\frac{\sin(3\lambda-2\Omega'-\omega-\Omega)}{3\dot{\lambda}'-2\dot{\Omega}'-\dot{\omega}-\dot{\Omega}}$$

$$-\frac{5}{8}\sin i\left(11+10\cos i-45\cos^2 i\right)\sin^2 i'\sin^2\frac{i'}{2}\;\frac{\sin(3\lambda'-4\Omega'-\omega+\Omega)}{3\dot{\lambda}'-4\dot{\Omega}'-\dot{\omega}+\dot{\Omega}}$$

$$+\frac{5}{2}\left(2+7\cos i-4\cos^2 i-9\cos^3 i\right)\sin i'\cos^4\frac{i'}{2}\;\frac{\sin(3\lambda'-\Omega'-\omega-2\Omega)}{3\dot{\lambda}'-\dot{\Omega}'-\dot{\omega}-2\dot{\Omega}}$$

$$-\frac{5}{2}\left(2-7\cos i-4\cos^2 i+9\cos^3 i\right)\sin i'\sin^4\frac{i'}{2}\;\frac{\sin(3\lambda'-5\Omega'-\omega+2\Omega)}{3\dot{\lambda}'-5\dot{\Omega}'-\dot{\omega}+2\dot{\Omega}}$$

$$-\frac{5}{2}\sin i\left(1-2\cos i-3\cos^2 i\right)\cos^6\frac{i'}{2}\;\frac{\sin(3\lambda'-\omega-3\Omega)}{3\dot{\lambda}'-\dot{\omega}-3\dot{\Omega}}$$

$$+\frac{5}{2}\sin i\left(1+2\cos i-3\cos^2 i\right)\sin^6\frac{i'}{2}\;\left.\frac{\sin(3\lambda'-6\Omega'-\omega+3\Omega)}{3\dot{\lambda}'-6\dot{\Omega}'-\dot{\omega}+3\dot{\Omega}}\right]$$

$$-\frac{175}{64}\,m'\,e^3\,\frac{a^3}{a'^4}\left[\frac{9}{16}\cos i\sin^2 i\sin i'\left(1-5\cos^2 i'\right)\frac{\sin(\lambda'-\Omega'+3\omega)}{\dot{\lambda}'-\dot{\Omega}'+3\dot{\omega}}\right.$$

$$+\frac{3}{32}\sin i\left(1-2\cos i-3\cos^2 i\right)\sin^2\frac{i'}{2}\left(1-10\cos i'-15\cos^2 i'\right)\frac{\sin(\lambda'-2\Omega'+3\omega+\Omega)}{\dot\lambda'-2\dot\Omega'+3\dot\omega+\dot\Omega}$$

$$-\frac{3}{32}\sin i\left(1+2\cos i-3\cos^2 i\right)\cos^2\frac{i'}{2}\left(1+10\cos i'-15\cos^2 i'\right)\frac{\sin(\lambda'+3\omega-\Omega)}{\dot\lambda'+3\dot\omega-\dot\Omega}$$

$$+\frac{3}{16}\cos^2\frac{i}{2}\left(2-\cos i-3\cos^2 i\right)\sin i'\left(1+2\cos i'-3\cos^2 i'\right)\frac{\sin(\lambda'-3\Omega'+3\omega+2\Omega)}{\dot\lambda'-3\dot\Omega'+3\dot\omega+2\dot\Omega}$$

$$-\frac{3}{16}\sin^2\frac{i}{2}\left(2+\cos i-3\cos^2 i\right)\sin i'\left(1-2\cos i'-3\cos^2 i'\right)\frac{\sin(\lambda'+\Omega'+3\omega-2\Omega)}{\dot\lambda'+\dot\Omega'+3\dot\omega-2\dot\Omega}$$

$$-\frac{9}{8}\sin i\cos^4\frac{i}{2}\sin^2 i'\sin^2\frac{i'}{2}\frac{\sin(\lambda'-4\Omega'+3\omega+3\Omega)}{\dot\lambda'-4\dot\Omega'+3\dot\omega+3\dot\Omega}$$

$$+\frac{9}{8}\sin i\sin^4\frac{i}{2}\sin^2 i'\cos^2\frac{i'}{2}\frac{\sin(\lambda'+2\Omega'+3\omega-3\Omega)}{\dot\lambda'+2\dot\Omega'+3\dot\omega-3\dot\Omega}$$

$$-\frac{9}{16}\cos i\sin^2 i\sin i'\left(1-5\cos^2 i'\right)\frac{\sin(\lambda'-\Omega'-3\omega)}{\dot\lambda'-\dot\Omega'-3\dot\omega}$$

$$+\frac{3}{32}\sin i\left(1-2\cos i-3\cos^2 i\right)\cos^2\frac{i'}{2}\left(1+10\cos i'-15\cos^2 i'\right)\frac{\sin(\lambda'-3\omega-\Omega)}{\dot\lambda'-3\dot\omega-\dot\Omega}$$

$$-\frac{3}{32}\sin i\left(1+2\cos i-3\cos^2 i\right)\sin^2\frac{i'}{2}\left(1-10\cos i'-15\cos^2 i'\right)\frac{\sin(\lambda'-2\Omega'-3\omega+\Omega)}{\dot\lambda'-2\dot\Omega'-3\dot\omega+\dot\Omega}$$

$$-\frac{3}{16}\cos^2\frac{i}{2}\left(2-\cos i-3\cos^2 i\right)\sin i'\left(1-2\cos i'-3\cos^2 i'\right)\frac{\sin(\lambda'+\Omega'-3\omega-2\Omega)}{\dot\lambda'+\dot\Omega'-3\dot\omega-2\dot\Omega}$$

$$+\frac{3}{16}\sin^2\frac{i}{2}\left(2+\cos i-3\cos^2 i\right)\sin i'\left(1+2\cos i'-3\cos^2 i'\right)\frac{\sin(\lambda'-3\Omega'-3\omega+2\Omega)}{\dot\lambda'-3\dot\Omega'-3\dot\omega+2\dot\Omega}$$

$$-\frac{9}{8}\sin i\cos^4\frac{i}{2}\sin^2 i'\cos^2\frac{i'}{2}\frac{\sin(\lambda'+2\Omega'-3\omega-3\Omega)}{\dot\lambda'+2\dot\Omega'-3\dot\omega-3\dot\Omega}$$

$$+\frac{9}{8}\sin i\sin^4\frac{i}{2}\sin^2 i'\sin^2\frac{i'}{2}\frac{\sin(\lambda'-4\Omega'-3\omega+3\Omega)}{\dot\lambda'-4\dot\Omega'-3\dot\omega+3\dot\Omega}$$

$$-\frac{15}{16}\cos i\sin^2 i\sin^3 i'\frac{\sin(3\lambda'-3\Omega'+3\omega)}{3\dot\lambda'-3\dot\Omega'+3\dot\omega}$$

$$- \frac{15}{32} \sin i \left(1 - 2\cos i - 3\cos^2 i\right) \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(3\lambda' - 4\Omega' + 3\omega + \Omega)}{3\dot{\lambda}' - 4\dot{\Omega}' + 3\dot{\omega} + \dot{\Omega}}$$

$$+ \frac{15}{32} \sin i \left(1 + 2\cos i - 3\cos^2 i\right) \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(3\lambda' - 2\Omega' + 3\omega - \Omega)}{3\dot{\lambda}' - 2\dot{\Omega}' + 3\dot{\omega} - \dot{\Omega}}$$

$$+ \frac{3}{4} \cos^2 \frac{i}{2} \left(2 - \cos i - 3\cos^2 i\right) \sin i' \sin^4 \frac{i'}{2} \frac{\sin(3\lambda' - 5\Omega' + 3\omega + 2\Omega)}{3\dot{\lambda}' - 5\dot{\Omega}' + 3\dot{\omega} + 2\dot{\Omega}}$$

$$- \frac{3}{4} \sin^2 \frac{i}{2} \left(2 + \cos i - 3\cos^2 i\right) \sin i' \cos^4 \frac{i'}{2} \frac{\sin(3\lambda' - \Omega' + 3\omega - 2\Omega)}{3\dot{\lambda}' - \dot{\Omega}' + 3\dot{\omega} - 2\dot{\Omega}}$$

$$- \frac{3}{2} \sin i \cos^4 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\sin(3\lambda' - 6\Omega' + 3\omega + 3\Omega)}{3\dot{\lambda}' - 6\dot{\Omega}' + 3\dot{\omega} + 3\dot{\Omega}}$$

$$+ \frac{3}{2} \sin i \sin^4 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\sin(3\lambda' + 3\omega - 3\Omega)}{3\dot{\lambda}' + 3\dot{\omega} - 3\dot{\Omega}}$$

$$+ \frac{15}{16} \cos i \sin^2 i \sin^3 i' \frac{\sin(3\lambda' - 3\Omega' - 3\omega)}{3\dot{\lambda}' - 3\dot{\Omega}' - 3\dot{\omega}}$$

$$- \frac{15}{32} \sin i \left(1 - 2\cos i - 3\cos^2 i\right) \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(3\lambda' - 2\Omega' - 3\omega - \Omega)}{3\dot{\lambda}' - 2\dot{\Omega}' - 3\dot{\omega} - \dot{\Omega}}$$

$$+ \frac{15}{32} \sin i \left(1 + 2\cos i - 3\cos^2 i\right) \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(3\lambda' - 4\Omega' - 3\omega + \Omega)}{3\dot{\lambda}' - 4\dot{\Omega}' - 3\dot{\omega} + \dot{\Omega}}$$

$$- \frac{3}{4} \cos^2 \frac{i}{2} \left(2 - \cos i - 3\cos^2 i\right) \sin i' \cos^4 \frac{i'}{2} \frac{\sin(3\lambda' - \Omega' - 3\omega - 2\Omega)}{3\dot{\lambda}' - \dot{\Omega}' - 3\dot{\omega} - 2\dot{\Omega}}$$

$$+ \frac{3}{4} \sin^2 \frac{i}{2} \left(2 + \cos i - 3\cos^2 i\right) \sin i' \sin^4 \frac{i'}{2} \frac{\sin(3\lambda' - 5\Omega' - 3\omega + 2\Omega)}{3\dot{\lambda}' - 5\dot{\Omega}' - 3\dot{\omega} + 2\dot{\Omega}}$$

$$- \frac{3}{2} \sin i \cos^4 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\sin(3\lambda' - 3\omega - 3\Omega)}{3\dot{\lambda}' - 3\dot{\omega} - 3\dot{\Omega}}$$

$$+ \frac{3}{2} \sin i \sin^4 \frac{i}{2} \sin^6 \frac{i'}{2} \left.\frac{\sin(3\lambda' - 6\Omega' - 3\omega + 3\Omega)}{3\dot{\lambda}' - 6\dot{\Omega}' - 3\dot{\omega} + 3\dot{\Omega}}\right] ;$$

$$A_6 = \int \delta e \, dt$$

$$= -\frac{\sqrt{1-e^2}}{na^2 e}\int A_1 \, dt$$

$$= \frac{15}{128}\frac{n'^2}{n} m' e\sqrt{1-e^2}\left\{8\cos^4\frac{i}{2}\cos^4\frac{i'}{2}\left[\frac{4\sin(2\lambda'-2\omega-2\Omega)}{4(\dot\lambda'-\dot\omega-\dot\Omega)^2} - 2e'\frac{\sin(\lambda'+\omega'+\Omega'-2\omega-2\Omega)}{(\dot\lambda'+\dot\omega'+\dot\Omega'-2\dot\omega-2\dot\Omega)^2}\right.\right.$$

$$\left. + 14e'\frac{\sin(3\lambda'-\omega'-\Omega'-2\omega-2\Omega)}{(3\dot\lambda'-\dot\omega'-\dot\Omega'-2\dot\omega-2\dot\Omega)^2}\right]$$

$$-4\sin^2 i\left(1-\frac{3}{2}\sin^2 i'\right)\left[\frac{4\sin 2\omega}{4\dot\omega^2} - 6e'\frac{\sin(\lambda'-\omega'-\Omega'-2\omega)}{(\dot\lambda'-\dot\omega'-\dot\Omega'-2\dot\omega)^2} + 6e'\frac{\sin(\lambda'-\omega'-\Omega'+2\omega)}{(\dot\lambda'-\dot\omega'-\dot\Omega'+2\dot\omega)^2}\right]$$

$$+4\cos^4\frac{i}{2}\sin^2 i'\left[\frac{4\sin(2\Omega'-2\omega-2\Omega)}{4(\dot\Omega'-\dot\omega-\dot\Omega)^2} + 6e'\frac{\sin(\lambda'-\omega'+\Omega'-2\omega-2\Omega)}{(\dot\lambda'-\dot\omega'+\dot\Omega'-2\dot\omega-2\dot\Omega)^2} - 6e'\frac{\sin(\lambda'-\omega'-3\Omega'+2\omega+2\Omega)}{(\dot\lambda'-\dot\omega'-3\dot\Omega'+2\dot\omega+2\dot\Omega)^2}\right]$$

$$-8\sin^4\frac{i}{2}\cos^4\frac{i'}{2}\left[\frac{4\sin(2\lambda'+2\omega-2\Omega)}{4(\dot\lambda'+\dot\omega-\dot\Omega)^2} - 2e'\frac{\sin(\lambda'+\omega'+\Omega'+2\omega-2\Omega)}{(\dot\lambda'+\dot\omega'+\dot\Omega'+2\dot\omega-2\dot\Omega)^2} + 14e'\frac{\sin(3\lambda'-\omega'-\Omega'+2\omega-2\Omega)}{(3\dot\lambda'-\dot\omega'-\dot\Omega'+2\dot\omega-2\dot\Omega)^2}\right]$$

$$-8\cos^4\frac{i}{2}\sin^4\frac{i'}{2}\left[\frac{4\sin(2\lambda'-4\Omega'+2\omega+2\Omega)}{4(\dot\lambda'-2\dot\Omega'+\dot\omega+\dot\Omega)^2} - 2e'\frac{\sin(\lambda'+\omega'-3\Omega'+2\omega+2\Omega)}{(\dot\lambda'+\dot\omega'-3\dot\Omega'+2\dot\omega+2\dot\Omega)^2} + 14e'\frac{\sin(3\lambda'-\omega'-5\Omega'+2\omega+2\Omega)}{(3\dot\lambda'-\dot\omega'-5\dot\Omega'+2\dot\omega+2\dot\Omega)^2}\right]$$

$$+8\sin^4\frac{i}{2}\sin^4\frac{i'}{2}\left[\frac{4\sin(2\lambda'-4\Omega'-2\omega+2\Omega)}{4(\dot\lambda'-2\dot\Omega'-\dot\omega+\dot\Omega)^2} - 2e'\frac{\sin(\lambda'+\omega'-3\Omega'-2\omega+2\Omega)}{(\dot\lambda'+\dot\omega'-3\dot\Omega'-2\dot\omega+2\dot\Omega)^2} + 14e'\frac{\sin(3\lambda'-\omega'-5\Omega'-2\omega+2\Omega)}{(3\dot\lambda'-\dot\omega'-5\dot\Omega'-2\dot\omega+2\dot\Omega)^2}\right]$$

$$+3\sin^2 i\sin^2 i'\left[\frac{4\sin(2\lambda'-2\Omega'-2\omega)}{4(\dot\lambda'-\dot\Omega'-\dot\omega)^2} - 2e'\frac{\sin(\lambda'+\omega'-\Omega'-2\omega)}{(\dot\lambda'+\dot\omega'-\dot\Omega'-2\dot\omega)^2} + 14e'\frac{\sin(3\lambda'-\omega'-3\Omega'-2\omega)}{(3\dot\lambda'-\dot\omega'-3\dot\Omega'-2\dot\omega)^2}\right]$$

$$-3\sin^2 i\sin^2 i'\left[\frac{4\sin(2\lambda'-2\Omega'+2\omega)}{4(\dot\lambda'-\dot\Omega'+\dot\omega)^2} - 2e'\frac{\sin(\lambda'+\omega'-\Omega'+2\omega)}{(\dot\lambda'+\dot\omega'-\dot\Omega'+2\dot\omega)^2} + 14e'\frac{\sin(3\lambda'-\omega'-3\Omega'+2\omega)}{(3\dot\lambda'-\dot\omega'-3\dot\Omega'+2\dot\omega)^2}\right]$$

$$+16\sin i\cos^2\frac{i}{2}\sin i'\cos^2\frac{i'}{2}\left[2\frac{\sin(2\lambda'-\Omega'-2\omega-\Omega)}{(2\dot\lambda'-\dot\Omega'-2\dot\omega-\dot\Omega)^2} - e'\frac{\sin(\lambda'+\omega'-2\omega-\Omega)}{(\dot\lambda'+\dot\omega'-2\dot\omega-\dot\Omega)^2} + 7e'\frac{\sin(3\lambda'-\omega'-2\Omega'-2\omega-\Omega)}{(3\dot\lambda'-\dot\omega'-2\dot\Omega'-2\dot\omega-\dot\Omega)^2}\right]$$

$$- 8 \sin i \cos^2 \frac{i}{2} \sin 2i' \left[2 \frac{\sin(\Omega' - 2\omega - \Omega)}{(\dot{\Omega}' - 2\dot{\omega} - \dot{\Omega})^2} + 3e' \frac{\sin(\lambda' - \omega' - 2\omega - \Omega)}{(\dot{\lambda}' - \dot{\omega}' - 2\dot{\omega} - \dot{\Omega})^2} - 3e' \frac{\sin(\lambda' - \omega' - 2\Omega' + 2\omega + \Omega)}{(\dot{\lambda}' - \dot{\omega}' - 2\dot{\Omega}' + 2\dot{\omega} + \dot{\Omega})^2}\right]$$

$$- 4 \sin^4 \frac{i}{2} \sin^2 i' \left[\frac{4 \sin(2\Omega' + 2\omega - 2\Omega)}{4(\dot{\Omega}' + \dot{\omega} - \dot{\Omega})^2} - 6e' \frac{\sin(\lambda' - \omega' - 3\Omega' - 2\omega + 2\Omega)}{(\dot{\lambda}' - \dot{\omega}' - 3\dot{\Omega}' - 2\dot{\omega} + 2\dot{\Omega})^2} + 6e' \frac{\sin(\lambda' - \omega' + \Omega' + 2\omega - 2\Omega)}{(\dot{\lambda}' - \dot{\omega}' + \dot{\Omega}' + 2\dot{\omega} - 2\dot{\Omega})^2}\right]$$

$$- 8 \sin i \sin^2 \frac{i}{2} \sin 2i' \left[2 \frac{\sin(\Omega' + 2\omega - \Omega)}{(\dot{\Omega}' + 2\dot{\omega} - \dot{\Omega})^2} - 3e' \frac{\sin(\lambda' - \omega' - 2\Omega' - 2\omega + \Omega)}{(\dot{\lambda}' - \dot{\omega}' - 2\dot{\Omega}' - 2\dot{\omega} + \dot{\Omega})^2} + 3e' \frac{\sin(\lambda' - \omega' + 2\omega - \Omega)}{(\dot{\lambda}' - \dot{\omega}' + 2\dot{\omega} - \dot{\Omega})^2}\right]$$

$$+ 16 \sin i \sin^2 \frac{i}{2} \sin i' \cos^2 \frac{i'}{2} \left[2 \frac{\sin(2\lambda' - \Omega' + 2\omega - \Omega)}{(2\dot{\lambda}' - \dot{\Omega}' + 2\dot{\omega} - \dot{\Omega})^2} - e' \frac{\sin(\lambda' + \omega' + 2\omega - \Omega)}{(\dot{\lambda}' + \dot{\omega}' + 2\dot{\omega} - \dot{\Omega})^2} + 7e' \frac{\sin(3\lambda' - \omega' - 2\Omega' + 2\omega - \Omega)}{(3\dot{\lambda}' - \dot{\omega}' - 2\dot{\Omega}' + 2\dot{\omega} - \dot{\Omega})^2}\right]$$

$$+ 16 \sin i \cos^2 \frac{i}{2} \sin i' \sin^2 \frac{i'}{2} \left[2 \frac{\sin(2\lambda' - 3\Omega' + 2\omega + \Omega)}{(2\dot{\lambda}' - 3\dot{\Omega}' + 2\dot{\omega} + \dot{\Omega})^2} - e' \frac{\sin(\lambda' + \omega' - 2\Omega' + 2\omega + \Omega)}{(\dot{\lambda}' + \dot{\omega}' - 2\dot{\Omega}' + 2\dot{\omega} + \dot{\Omega})^2} + 7e' \frac{\sin(3\lambda' - \omega' - 4\Omega' + 2\omega + \Omega)}{(3\dot{\lambda}' - \dot{\omega}' - 4\dot{\Omega}' + 2\dot{\omega} + \dot{\Omega})^2}\right]$$

$$+ 16 \sin i \sin^2 \frac{i}{2} \sin i' \sin^2 \frac{i'}{2} \left[2 \frac{\sin(2\lambda' - 3\Omega' - 2\omega + \Omega)}{(2\dot{\lambda}' - 3\dot{\Omega}' - 2\dot{\omega} + \dot{\Omega})^2} - e' \frac{\sin(\lambda' + \omega' - 2\Omega' - 2\omega + \Omega)}{(\dot{\lambda}' + \dot{\omega}' - 2\dot{\Omega}' - 2\dot{\omega} + \dot{\Omega})^2} + 7e' \frac{\sin(3\lambda' - \omega' - 4\Omega' - 2\omega + \Omega)}{(3\dot{\lambda}' - \dot{\omega}' - 4\dot{\Omega}' - 2\dot{\omega} + \dot{\Omega})^2}\right]\Bigg\}$$

$$+ \left(\frac{m'}{n}\right)\left(\frac{a}{a'^4}\right)\left(\frac{15}{64} + \frac{45}{256} e^2\right)\sqrt{1 - e^2} \Bigg[- \frac{3}{4} \sin i \left(1 - 5\cos^2 i\right) \sin i' \left(1 - 5\cos^2 i'\right) \frac{\sin(\lambda' - \Omega' + \omega)}{\left(\dot{\lambda}' - \dot{\Omega}' + \dot{\omega}\right)^2}$$

$$+ \frac{1}{4} \cos^2 \frac{i}{2} \left(1 + 10\cos i - 15\cos^2 i\right) \sin^2 \frac{i'}{2} \left(1 - 10\cos i' - 15\cos^2 i'\right) \frac{\sin(\lambda' - 2\Omega' + \omega + \Omega)}{\left(\dot{\lambda}' - 2\dot{\Omega}' + \dot{\omega} + \dot{\Omega}\right)^2}$$

$$+ \frac{1}{4} \sin^2 \frac{i}{2} \left(1 - 10\cos i - 15\cos^2 i\right) \cos^2 \frac{i'}{2} \left(1 + 10\cos i' - 15\cos^2 i'\right) \frac{\sin(\lambda' + \omega - \Omega)}{\left(\dot{\lambda}' + \dot{\omega} - \dot{\Omega}\right)^2}$$

$$- \frac{5}{8} \sin i \left(1 - 2\cos i - 3\cos^2 i\right) \sin i' \left(1 + 2\cos i' - 3\cos^2 i'\right) \frac{\sin(\lambda' - 3\Omega' + \omega + 2\Omega)}{\left(\dot{\lambda}' - 3\dot{\Omega}' + \dot{\omega} + 2\dot{\Omega}\right)^2}$$

$$- \frac{5}{8} \sin i \left(1 + 2\cos i - 3\cos^2 i\right) \sin i' \left(1 - 2\cos i' - 3\cos^2 i'\right) \frac{\sin(\lambda' + \Omega' + \omega - 2\Omega)}{\left(\dot{\lambda}' + \dot{\Omega}' + \dot{\omega} - 2\dot{\Omega}\right)^2}$$

$$+ \frac{15}{4} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(\lambda' - 4\Omega' + \omega + 3\Omega)}{\left(\dot{\lambda}' - 4\dot{\Omega}' + \dot{\omega} + 3\dot{\Omega}\right)^2}$$

$$+\frac{15}{4}\sin^2 i \sin^2\frac{i}{2}\sin^2 i'\cos^2\frac{i'}{2}\,\frac{\sin(\lambda'+2\Omega'+\omega-3\Omega)}{(\dot\lambda'+2\dot\Omega'+\dot\omega-3\dot\Omega)^2}$$

$$-\frac{3}{4}\sin i\,(1-5\cos^2 i)\sin i'\,(1-5\cos^2 i')\,\frac{\sin(\lambda'-\Omega'-\omega)}{(\dot\lambda'-\dot\Omega'-\dot\omega)^2}$$

$$-\frac{1}{4}\cos^2\frac{i}{2}\,(1+10\cos i-15\cos^2 i)\cos^2\frac{i'}{2}\,(1+10\cos i'-15\cos^2 i')\,\frac{\sin(\lambda'-\omega-\Omega)}{(\dot\lambda'-\dot\omega-\dot\Omega)^2}$$

$$-\frac{1}{4}\sin^2\frac{i}{2}\,(1-10\cos i-15\cos^2 i)\sin^2\frac{i'}{2}\,(1-10\cos i'-15\cos^2 i')\,\frac{\sin(\lambda'-2\Omega'-\omega+\Omega)}{(\dot\lambda'-2\dot\Omega'-\dot\omega+\dot\Omega)^2}$$

$$-\frac{5}{8}\sin i\,(1-2\cos i-3\cos^2 i)\sin i'\,(1-2\cos i'-3\cos^2 i')\,\frac{\sin(\lambda'+\Omega'-\omega-2\Omega)}{(\dot\lambda'+\dot\Omega'-\dot\omega-2\dot\Omega)^2}$$

$$-\frac{5}{8}\sin i\,(1+2\cos i-3\cos^2 i)\sin i'\,(1+2\cos i'-3\cos^2 i')\,\frac{\sin(\lambda'-3\Omega'-\omega+2\Omega)}{(\dot\lambda'-3\dot\Omega'-\dot\omega+2\dot\Omega)^2}$$

$$-\frac{15}{4}\sin^2 i\cos^2\frac{i}{2}\sin^2 i'\cos^2\frac{i'}{2}\,\frac{\sin(\lambda'+2\Omega'-\omega-3\Omega)}{(\dot\lambda'+2\dot\Omega'-\dot\omega-3\dot\Omega)^2}$$

$$-\frac{15}{4}\sin^2 i\sin^2\frac{i}{2}\sin^2 i'\sin^2\frac{i'}{2}\,\frac{\sin(\lambda'-4\Omega'-\omega+3\Omega)}{(\dot\lambda'-4\dot\Omega'-\dot\omega+3\dot\Omega)^2}$$

$$+\frac{5}{4}\sin i\,(1-5\cos^2 i)\sin^3 i'\,\frac{\sin(3\lambda'-3\Omega'+\omega)}{(3\dot\lambda'-3\dot\Omega'+\dot\omega)^2}$$

$$-\frac{5}{4}\cos^2\frac{i}{2}\,(1+10\cos i-15\cos^2 i)\sin^2 i'\sin^2\frac{i'}{2}\,\frac{\sin(3\lambda'-4\Omega'+\omega+\Omega)}{(3\dot\lambda'-4\dot\Omega'+\dot\omega+\dot\Omega)^2}$$

$$-\frac{5}{4}\sin^2\frac{i}{2}\,(1-10\cos i-15\cos^2 i)\sin^2 i'\cos^2\frac{i'}{2}\,\frac{\sin(3\lambda'-2\Omega'+\omega-\Omega)}{(3\dot\lambda'-2\dot\Omega'+\dot\omega-\dot\Omega)^2}$$

$$-\frac{5}{2}\sin i\,(1-2\cos i-3\cos^2 i)\sin i'\sin^4\frac{i'}{2}\,\frac{\sin(3\lambda'-5\Omega'+\omega+2\Omega)}{(3\dot\lambda'-5\dot\Omega'+\dot\omega+2\dot\Omega)^2}$$

$$- \frac{5}{2} \sin i \left(1 + 2\cos i - 3\cos^2 i\right) \sin i' \cos^4 \frac{i'}{2} \frac{\sin(3\lambda' - \Omega' + \omega - 2\Omega)}{\left(3\dot{\lambda}' - \dot{\Omega}' + \dot{\omega} - 2\dot{\Omega}\right)^2}$$

$$+ 5\sin^2 i \cos^2 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\sin(3\lambda' - 6\Omega' + \omega + 3\Omega)}{\left(3\dot{\lambda}' - 6\dot{\Omega}' + \dot{\omega} + 3\dot{\Omega}\right)^2}$$

$$+ 5\sin^2 i \sin^2 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\sin(3\lambda' + \omega - 3\Omega)}{\left(3\dot{\lambda}' + \dot{\omega} - 3\dot{\Omega}\right)^2}$$

$$+ \frac{5}{4} \sin i \left(1 - 5\cos^2 i\right) \sin^3 i' \frac{\sin(3\lambda' - 3\Omega' - \omega)}{\left(3\dot{\lambda}' - 3\dot{\Omega}' - \dot{\omega}\right)^2}$$

$$+ \frac{5}{4} \cos^2 \frac{i}{2} \left(1 + 10\cos i - 15\cos^2 i\right) \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(3\lambda' - 2\Omega' - \omega - \Omega)}{\left(3\dot{\lambda}' - 2\dot{\Omega}' - \dot{\omega} - \dot{\Omega}\right)^2}$$

$$+ \frac{5}{4} \sin^2 \frac{i}{2} \left(1 - 10\cos i - 15\cos^2 i\right) \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(3\lambda' - 4\Omega' - \omega + \Omega)}{\left(3\dot{\lambda}' - 4\dot{\Omega}' - \dot{\omega} + \dot{\Omega}\right)^2}$$

$$- \frac{5}{2} \sin i \left(1 - 2\cos i - 3\cos^2 i\right) \sin i' \cos^4 \frac{i'}{2} \frac{\sin(3\lambda' - \Omega' - \omega - 2\Omega)}{\left(3\dot{\lambda}' - \dot{\Omega}' - \dot{\omega} - 2\dot{\Omega}\right)^2}$$

$$- \frac{5}{2} \sin i \left(1 + 2\cos i - 3\cos^2 i\right) \sin i' \sin^4 \frac{i'}{2} \frac{\sin(3\lambda' - 5\Omega' - \omega + 2\Omega)}{\left(3\dot{\lambda}' - 5\dot{\Omega}' - \dot{\omega} + 2\dot{\Omega}\right)^2}$$

$$- 5\sin^2 i \cos^2 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\sin(3\lambda' - \omega - 3\Omega)}{\left(3\dot{\lambda}' - \dot{\omega} - 3\dot{\Omega}\right)^2}$$

$$- 5\sin^2 i \sin^2 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\sin(3\lambda' - 6\Omega' - \omega + 3\Omega)}{\left(3\dot{\lambda}' - 6\dot{\Omega}' - \dot{\omega} + 3\dot{\Omega}\right)^2} \Bigg]$$

$$+ \left(\frac{m'}{n}\right)\left(\frac{a}{a'^4}\right)\left(\frac{175}{64} e^2\right) \sqrt{1 - e^2} \Bigg[\frac{9}{16} \sin^3 i \sin i' \left(1 - 5\cos^2 i'\right) \frac{\sin(\lambda' - \Omega' + 3\omega)}{\left(\dot{\lambda}' - \dot{\Omega}' + 3\dot{\omega}\right)^2}$$

$$- \frac{9}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 \frac{i'}{2} \left(1 - 10\cos i' - 15\cos^2 i'\right) \frac{\sin(\lambda' - 2\Omega' + 3\omega + \Omega)}{\left(\dot{\lambda}' - 2\dot{\Omega}' + 3\dot{\omega} + \dot{\Omega}\right)^2}$$

$$- \frac{9}{16} \sin^2 i \sin^2 \frac{i}{2} \cos^2 \frac{i'}{2} \left(1 + 10 \cos i' - 15 \cos^2 i'\right) \frac{\sin(\lambda' + 3\omega - \Omega)}{\left(\dot{\lambda}' + 3\dot{\omega} - \dot{\Omega}\right)^2}$$

$$- \frac{9}{8} \sin i \cos^4 \frac{i}{2} \sin i' \left(1 + 2 \cos i' - 3 \cos^2 i'\right) \frac{\sin(\lambda' - 3\Omega' + 3\omega + 2\Omega)}{\left(\dot{\lambda}' - 3\dot{\Omega}' + 3\dot{\omega} + 2\dot{\Omega}\right)^2}$$

$$- \frac{9}{8} \sin i \sin^4 \frac{i}{2} \sin i' \left(1 - 2 \cos i' - 3 \cos^2 i'\right) \frac{\sin(\lambda' + \Omega' + 3\omega - 2\Omega)}{\left(\dot{\lambda}' + \dot{\Omega}' + 3\dot{\omega} - 2\dot{\Omega}\right)^2}$$

$$+ \frac{9}{4} \cos^6 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(\lambda' - 4\Omega' + 3\omega + 3\Omega)}{\left(\dot{\lambda}' - 4\dot{\Omega}' + 3\dot{\omega} + 3\dot{\Omega}\right)^2}$$

$$+ \frac{9}{4} \sin^6 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(\lambda' + 2\Omega' + 3\omega - 3\Omega)}{\left(\dot{\lambda}' + 2\dot{\Omega}' + 3\dot{\omega} - 3\dot{\Omega}\right)^2}$$

$$+ \frac{9}{16} \sin^3 i \sin i' \left(1 - 5 \cos^2 i'\right) \frac{\sin(\lambda' - \Omega' - 3\omega)}{\left(\dot{\lambda}' - \dot{\Omega}' - 3\dot{\omega}\right)^2}$$

$$+ \frac{9}{16} \sin^2 i \cos^2 \frac{i}{2} \cos^2 \frac{i'}{2} \left(1 + 10 \cos i' - 15 \cos^2 i'\right) \frac{\sin(\lambda' - 3\omega - \Omega)}{\left(\dot{\lambda}' - 3\dot{\omega} - \dot{\Omega}\right)^2}$$

$$+ \frac{9}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 \frac{i'}{2} \left(1 - 10 \cos i' - 15 \cos^2 i'\right) \frac{\sin(\lambda' - 2\Omega' - 3\omega + \Omega)}{\left(\dot{\lambda}' - 2\dot{\Omega}' - 3\dot{\omega} + \dot{\Omega}\right)^2}$$

$$- \frac{9}{8} \sin i \cos^4 \frac{i}{2} \sin i' \left(1 - 2 \cos i' - 3 \cos^2 i'\right) \frac{\sin(\lambda' + \Omega' - 3\omega - 2\Omega)}{\left(\dot{\lambda}' + \dot{\Omega}' - 3\dot{\omega} - 2\dot{\Omega}\right)^2}$$

$$- \frac{9}{8} \sin i \sin^4 \frac{i}{2} \sin i' \left(1 + 2 \cos i' - 3 \cos^2 i'\right) \frac{\sin(\lambda' - 3\Omega' - 3\omega + 2\Omega)}{\left(\dot{\lambda}' - 3\dot{\Omega}' - 3\dot{\omega} + 2\dot{\Omega}\right)^2}$$

$$- \frac{9}{4} \cos^6 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(\lambda' + 2\Omega' - 3\omega - 3\Omega)}{\left(\dot{\lambda}' + 2\dot{\Omega}' - 3\dot{\omega} - 3\dot{\Omega}\right)^2}$$

$$- \frac{9}{4} \sin^6 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(\lambda' - 4\Omega' - 3\omega + 3\Omega)}{\left(\dot{\lambda}' - 4\dot{\Omega}' - 3\dot{\omega} + 3\dot{\Omega}\right)^2}$$

$$- \frac{15}{16} \sin^3 i \sin^3 i' \frac{\sin(3\lambda' - 3\Omega' + 3\omega)}{\left(3\dot{\lambda}' - 3\dot{\Omega}' + 3\dot{\omega}\right)^2}$$

$$+ \frac{45}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin (3\lambda' - 4\Omega' + 3\omega + \Omega)}{(3\dot{\lambda}' - 4\dot{\Omega}' + 3\dot{\omega} + \dot{\Omega})^2}$$

$$+ \frac{45}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin (3\lambda' - 2\Omega' + 3\omega - \Omega)}{(3\dot{\lambda}' - 2\dot{\Omega}' + 3\dot{\omega} - \dot{\Omega})^2}$$

$$- \frac{9}{2} \sin i \cos^4 \frac{i}{2} \sin i' \sin^4 \frac{i'}{2} \frac{\sin (3\lambda' - 5\Omega' + 3\omega + 2\Omega)}{(3\dot{\lambda}' - 5\dot{\Omega}' + 3\dot{\omega} + 2\dot{\Omega})^2}$$

$$- \frac{9}{2} \sin i \sin^4 \frac{i}{2} \sin i' \cos^4 \frac{i'}{2} \frac{\sin (3\lambda' - \Omega' + 3\omega - 2\Omega)}{(3\dot{\lambda}' - \dot{\Omega}' + 3\dot{\omega} - 2\dot{\Omega})^2}$$

$$+ 3 \cos^6 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\sin (3\lambda' - 6\Omega' + 3\omega + 3\Omega)}{(3\dot{\lambda}' - 6\dot{\Omega}' + 3\dot{\omega} + 3\dot{\Omega})^2}$$

$$+ 3 \sin^6 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\sin (3\lambda' + 3\omega - 3\Omega)}{(3\dot{\lambda}' + 3\dot{\omega} - 3\dot{\Omega})^2}$$

$$- \frac{15}{16} \sin^3 i \sin^3 i' \frac{\sin (3\lambda' - 3\Omega' - 3\omega)}{(3\dot{\lambda}' - 3\dot{\Omega}' - 3\dot{\omega})^2}$$

$$- \frac{45}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin (3\lambda' - 2\Omega' - 3\omega - \Omega)}{(3\dot{\lambda}' - 2\dot{\Omega}' - 3\dot{\omega} - \dot{\Omega})^2}$$

$$- \frac{45}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin (3\lambda' - 4\Omega' - 3\omega + \Omega)}{(3\dot{\lambda}' - 4\dot{\Omega}' - 3\dot{\omega} + \dot{\Omega})^2}$$

$$- \frac{9}{2} \sin i \cos^4 \frac{i}{2} \sin i \cos^4 \frac{i'}{2} \frac{\sin (3\lambda' - \Omega' - 3\omega - 2\Omega)}{(3\dot{\lambda}' - \dot{\Omega}' - 3\dot{\omega} - 2\dot{\Omega})^2}$$

$$- \frac{9}{2} \sin i \sin^4 \frac{i}{2} \sin i' \sin^4 \frac{i'}{2} \frac{\sin (3\lambda' - 5\Omega' - 3\omega + 2\Omega)}{(3\dot{\lambda}' - 5\dot{\Omega}' - 3\dot{\omega} + 2\dot{\Omega})^2}$$

$$- 3 \cos^6 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\sin (3\lambda' - 3\omega - 3\Omega)}{(3\dot{\lambda}' - 3\dot{\omega} - 3\dot{\Omega})^2}$$

$$\left. - 3 \sin^6 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\sin (3\lambda' - 6\Omega' - 3\omega + 3\Omega)}{(3\dot{\lambda}' - 6\dot{\Omega}' - 3\dot{\omega} + 3\dot{\Omega})^2} \right] ;$$

and

$$A_7 = \int \delta i \, dt$$

$$= \frac{\cos i}{na^2 \sqrt{1-e^2} \sin i} \int A_1 \, dt - \frac{1}{na^2 \sqrt{1-e^2} \sin i} \int A_2 \, dt$$

$$= -\frac{e \cot i}{1-e^2} A_6$$

$$-\frac{3}{32} \frac{n'^2}{n} \frac{m'}{\sqrt{1-e^2} \sin i} \left(\left(1+\frac{3}{2}e^2\right) \left\{ -\sin 2i \sin 2i' \left[2 \frac{\sin(\Omega'-\Omega)}{(\dot{\Omega}'-\dot{\Omega})^2} + 3e' \frac{\sin(\lambda'-\omega'-\Omega)}{(\dot{\lambda}'-\dot{\omega}'-\dot{\Omega})^2} - 3e' \frac{\sin(\lambda'-\omega'-2\Omega'+\Omega)}{(\dot{\lambda}'-\dot{\omega}'-2\dot{\Omega}'+\dot{\Omega})^2} \right] \right. \right.$$

$$- \sin^2 i \sin^2 i' \left[\frac{4\sin(2\Omega'-2\Omega)}{4(\dot{\Omega}'-\dot{\Omega})^2} + 6e' \frac{\sin(\lambda'-\omega'+\Omega'-2\Omega)}{(\dot{\lambda}'-\dot{\omega}'+\dot{\Omega}'-2\dot{\Omega})^2} - 6e' \frac{\sin(\lambda'-\omega'-3\Omega'+2\Omega)}{(\dot{\lambda}'-\dot{\omega}'-3\dot{\Omega}'+2\dot{\Omega})^2} \right]$$

$$- 2\sin^2 i \cos^4 \frac{i'}{2} \left[\frac{4\sin(2\lambda'-2\Omega)}{4(\dot{\lambda}'-\dot{\Omega})^2} - 2e' \frac{\sin(\lambda'+\omega'+\Omega'-2\Omega)}{(\dot{\lambda}'+\dot{\omega}'+\dot{\Omega}'-2\dot{\Omega})^2} + 14e' \frac{\sin(3\lambda'-\omega'-\Omega'-2\Omega)}{(3\dot{\lambda}'-\dot{\omega}'-\dot{\Omega}'-2\dot{\Omega})^2} \right]$$

$$+ 2\sin 2i \sin i' \cos^2 \frac{i'}{2} \left[2\frac{\sin(2\lambda'-\Omega'-\Omega)}{(2\dot{\lambda}'-\dot{\Omega}'-\dot{\Omega})^2} - e' \frac{\sin(\lambda'+\omega'-\Omega)}{(\dot{\lambda}'+\dot{\omega}'-\dot{\Omega})^2} + 7e' \frac{\sin(3\lambda'-\omega'-2\Omega'-\Omega)}{(3\dot{\lambda}'-\dot{\omega}'-2\dot{\Omega}'-\dot{\Omega})^2} \right]$$

$$+ 2\sin^2 i \sin^4 \frac{i'}{2} \left[\frac{4\sin(2\lambda'-4\Omega'+2\Omega)}{4(\dot{\lambda}'-2\dot{\Omega}'+\dot{\Omega})^2} - 2e' \frac{\sin(\lambda'+\omega'-3\Omega'+2\Omega)}{(\dot{\lambda}'+\dot{\omega}'-3\dot{\Omega}'+2\dot{\Omega})^2} + 14e' \frac{\sin(3\lambda'-\omega'-5\Omega'+2\Omega)}{(3\dot{\lambda}'-\dot{\omega}'-5\dot{\Omega}'+2\dot{\Omega})^2} \right]$$

$$\left. + 2\sin 2i \sin i' \sin^2 \frac{i'}{2} \left[2\frac{\sin(2\lambda'-3\Omega'+\Omega)}{(2\dot{\lambda}'-3\dot{\Omega}'+\dot{\Omega})^2} - e' \frac{\sin(\lambda'+\omega'-2\Omega'+\Omega)}{(\dot{\lambda}'+\dot{\omega}'-2\dot{\Omega}'+\dot{\Omega})^2} + 7e' \frac{\sin(3\lambda'-\omega'-4\Omega'+\Omega)}{(3\dot{\lambda}'-\dot{\omega}'-4\dot{\Omega}'+\dot{\Omega})^2} \right] \right\}$$

$$- 5e^2 \left\{ 2\cos^4 \frac{i}{2} \cos^4 \frac{i'}{2} \left[\frac{4\sin(2\lambda'-2\omega-2\Omega)}{4(\dot{\lambda}'-\dot{\omega}-\dot{\Omega})^2} - 2e' \frac{\sin(\lambda'+\omega'+\Omega'-2\omega-2\Omega)}{(\dot{\lambda}'+\dot{\omega}'+\dot{\Omega}'-2\dot{\omega}-2\dot{\Omega})^2} + 14e' \frac{\sin(3\lambda'-\omega'-\Omega'-2\omega-2\Omega)}{(3\dot{\lambda}'-\dot{\omega}'-\dot{\Omega}'-2\dot{\omega}-2\dot{\Omega})^2} \right] \right.$$

$$+ \cos^4 \frac{i}{2} \sin^2 i' \left[\frac{4\cos(2\Omega'-2\omega-2\Omega)}{4(\dot{\Omega}'-\dot{\omega}-\dot{\Omega})^2} + 6e' \frac{\sin(\lambda'-\omega'+\Omega'-2\omega-2\Omega)}{(\dot{\lambda}'-\dot{\omega}'+\dot{\Omega}'-2\dot{\omega}-2\dot{\Omega})^2} - 6e' \frac{\sin(\lambda'-\omega'-3\Omega'+2\omega+2\Omega)}{(\dot{\lambda}'-\dot{\omega}'-3\dot{\Omega}'+2\dot{\omega}+2\dot{\Omega})^2} \right]$$

$$+ 2\sin^4 \frac{i}{2} \cos^4 \frac{i'}{2} \left[\frac{4\sin(2\lambda'+2\omega-2\Omega)}{4(\dot{\lambda}'+\dot{\omega}-\dot{\Omega})^2} - 2e' \frac{\sin(\lambda'+\omega'+\Omega'+2\omega-2\Omega)}{(\dot{\lambda}'+\dot{\omega}'+\dot{\Omega}'+2\dot{\omega}-2\dot{\Omega})^2} + 14e' \frac{\sin(3\lambda'-\omega'-\Omega'+2\omega-2\Omega)}{(3\dot{\lambda}'-\dot{\omega}'-\dot{\Omega}'+2\dot{\omega}-2\dot{\Omega})^2} \right]$$

$$- 2\cos^4 \frac{i}{2} \sin^4 \frac{i'}{2} \left[\frac{4\sin(2\lambda'-4\Omega'+2\omega+2\Omega)}{4(\dot{\lambda}'-2\dot{\Omega}'+\dot{\omega}+\dot{\Omega})^2} - 2e' \frac{\sin(\lambda'+\omega'-3\Omega'+2\omega+2\Omega)}{(\dot{\lambda}'+\dot{\omega}'-3\dot{\Omega}'+2\dot{\omega}+2\dot{\Omega})^2} + 14e' \frac{\sin(3\lambda'-\omega'-5\Omega'+2\omega+2\Omega)}{(3\dot{\lambda}'-\dot{\omega}'-5\dot{\Omega}'+2\dot{\omega}+2\dot{\Omega})^2} \right]$$

$$-2\sin^4\frac{i}{2}\sin^4\frac{i'}{2}\left[\frac{4\sin(2\lambda'-4\Omega'-2\omega+2\Omega)}{4(\dot\lambda'-2\dot\Omega'-\dot\omega+\dot\Omega)^2}-2e'\frac{\sin(\lambda'+\omega'-3\Omega'-2\omega+2\Omega)}{(\dot\lambda'+\dot\omega'-3\dot\Omega'-2\dot\omega+2\dot\Omega)^2}+14e'\frac{\sin(3\lambda'-\omega'-5\Omega'-2\omega+2\Omega)}{(3\dot\lambda'-\dot\omega'-5\dot\Omega'-2\dot\omega+2\dot\Omega)^2}\right]$$

$$+2\sin i\cos^2\frac{i}{2}\sin i'\cos^2\frac{i'}{2}\left[2\frac{\sin(2\lambda'-\Omega'-2\omega-\Omega)}{(2\dot\lambda'-\dot\Omega'-2\dot\omega-\dot\Omega)^2}-e'\frac{\sin(\lambda'+\omega'-2\omega-\Omega)}{(\dot\lambda'+\dot\omega'-2\dot\omega-\dot\Omega)^2}+7e'\frac{\sin(3\lambda'-\omega'-2\Omega'-2\omega-\Omega)}{(3\dot\lambda'-\dot\omega'-2\dot\Omega'-2\dot\omega-\dot\Omega)^2}\right]$$

$$-\sin i\cos^2\frac{i}{2}\sin 2i'\left[2\frac{\sin(\Omega'-2\omega-\Omega)}{(\dot\Omega'-2\dot\omega-\dot\Omega)^2}+3e'\frac{\sin(\lambda'-\omega'-2\omega-\Omega)}{(\dot\lambda'-\dot\omega'-2\dot\omega-\dot\Omega)^2}-3e'\frac{\sin(\lambda'-\omega'-2\Omega'+2\omega+\Omega)}{(\dot\lambda'-\dot\omega'-2\dot\Omega'+2\dot\omega+\dot\Omega)^2}\right]$$

$$+\sin^4\frac{i}{2}\sin^2 i'\left[\frac{4\sin(2\Omega'+2\omega-2\Omega)}{4(\dot\Omega'+\dot\omega-\dot\Omega)^2}-6e'\frac{\sin(\lambda'-\omega'-3\Omega'-2\omega+2\Omega)}{(\dot\lambda'-\dot\omega'-3\dot\Omega'-2\dot\omega+2\dot\Omega)^2}+6e'\frac{\sin(\lambda'-\omega'+\Omega'+2\omega-2\Omega)}{(\dot\lambda'-\dot\omega'+\dot\Omega'+2\dot\omega-2\dot\Omega)^2}\right]$$

$$+\sin i\sin^2\frac{i}{2}\sin 2i'\left[2\frac{\sin(\Omega'+2\omega-\Omega)}{(\dot\Omega'+2\dot\omega-\dot\Omega)^2}-3e'\frac{\sin(\lambda'-\omega'-2\Omega'-2\omega+\Omega)}{(\dot\lambda'-\dot\omega'-2\dot\Omega'-2\dot\omega+\dot\Omega)^2}+3e'\frac{\sin(\lambda'-\omega'+2\omega-\Omega)}{(\dot\lambda'-\dot\omega'+2\dot\omega-\dot\Omega)^2}\right]$$

$$-2\sin i\sin^2\frac{i}{2}\sin i'\cos^2\frac{i'}{2}\left[2\frac{\sin(2\lambda'-\Omega'+2\omega-\Omega)}{(2\dot\lambda'-\dot\Omega'+2\dot\omega-\dot\Omega)^2}-e'\frac{\sin(\lambda'+\omega'+2\omega-\Omega)}{(\dot\lambda'+\dot\omega'+2\dot\omega-\dot\Omega)^2}+7e'\frac{\sin(3\lambda'-\omega'-2\Omega'+2\omega-\Omega)}{(3\dot\lambda'-\dot\omega'-2\dot\Omega'+2\dot\omega-\dot\Omega)^2}\right]$$

$$+2\sin i\cos^2\frac{i}{2}\sin i'\sin^2\frac{i'}{2}\left[2\frac{\sin(2\lambda'-3\Omega'+2\omega+\Omega)}{(2\dot\lambda'-3\dot\Omega'+2\dot\omega+\dot\Omega)^2}-e'\frac{\sin(\lambda'+\omega'-2\Omega'+2\omega+\Omega)}{(\dot\lambda'+\dot\omega'-2\dot\Omega'+2\dot\omega+\dot\Omega)^2}+7e'\frac{\sin(3\lambda'-\omega'-4\Omega'+2\omega+\Omega)}{(3\dot\lambda'-\dot\omega'-4\dot\Omega'+2\dot\omega+\dot\Omega)^2}\right]$$

$$-2\sin i\sin^2\frac{i}{2}\sin i'\sin^2\frac{i'}{2}\left[2\frac{\sin(2\lambda'-3\Omega'-2\omega+\Omega)}{(2\dot\lambda'-3\dot\Omega'-2\dot\omega+\dot\Omega)^2}-e'\frac{\sin(\lambda'+\omega'-2\Omega'-2\omega+\Omega)}{(\dot\lambda'+\dot\omega'-2\dot\Omega'-2\dot\omega+\dot\Omega)^2}+7e'\frac{\sin(3\lambda'-\omega'-4\Omega'-2\omega+\Omega)}{(3\dot\lambda-\dot\omega'-4\dot\Omega'-2\dot\omega+\dot\Omega)^2}\right]\bigg]\bigg\}\bigg).$$

$$+\left(\frac{m'}{n}\right)\left(\frac{a}{a'^4}\right)\frac{1}{\sqrt{1-e^2}\sin i}\left(\frac{15}{64}e+\frac{45}{256}e^3\right)\bigg[\frac{1}{4}\cos^2\frac{i}{2}\left(1+10\cos i-15\cos^2 i\right)\sin^2\frac{i'}{2}\left(1-10\cos i'-15\cos^2 i'\right)\frac{\sin(\lambda'-2\Omega'+\omega+\Omega)}{(\dot\lambda'-2\dot\Omega'+\dot\omega+\dot\Omega)^2}$$

$$-\frac{1}{4}\sin^2\frac{i}{2}\left(1-10\cos i-15\cos^2 i\right)\cos^2\frac{i'}{2}\left(1+10\cos i'-15\cos^2 i'\right)\frac{\sin(\lambda'+\omega-\Omega)}{(\dot\lambda'+\dot\omega-\dot\Omega)^2}$$

$$-\frac{5}{4}\sin i\left(1-2\cos i-3\cos^2 i\right)\sin i'\left(1+2\cos i'-3\cos^2 i'\right)\frac{\sin(\lambda'-3\Omega'+\omega+2\Omega)}{(\dot\lambda'-3\dot\Omega'+\dot\omega+2\dot\Omega)^2}$$

$$+\frac{5}{4}\sin i\left(1+2\cos i-3\cos^2 i\right)\sin i'\left(1-2\cos i'-3\cos^2 i'\right)\frac{\sin(\lambda'+\Omega'+\omega-2\Omega)}{(\dot\lambda'+\dot\Omega'+\dot\omega-2\dot\Omega)^2}$$

$$+\frac{45}{4}\sin^2 i\cos^2\frac{i}{2}\sin^2 i'\sin^2\frac{i'}{2}\frac{\sin(\lambda'-4\Omega'+\omega+3\Omega)}{(\dot\lambda'-4\dot\Omega'+\dot\omega+3\dot\Omega)^2}$$

$$-\frac{45}{4}\sin^2 i\sin^2\frac{i}{2}\sin^2 i'\cos^2\frac{i'}{2}\frac{\sin(\lambda'+2\Omega'+\omega-3\Omega)}{(\dot\lambda'+2\dot\Omega'+\dot\omega-3\dot\Omega)^2}$$

$$-\frac{1}{4}\cos^2\frac{i}{2}\left(1+10\cos i-15\cos^2 i\right)\cos^2\frac{i'}{2}\left(1+10\cos i'-15\cos^2 i'\right)\frac{\sin(\lambda'-\omega-\Omega)}{\left(\dot{\lambda}'-\dot{\omega}-\dot{\Omega}\right)^2}$$

$$+\frac{1}{4}\sin^2\frac{i}{2}\left(1-10\cos i-15\cos^2 i\right)\sin^2\frac{i'}{2}\left(1-10\cos i-15\cos^2 i\right)\frac{\sin(\lambda'-2\Omega'-\omega+\Omega)}{(\dot{\lambda}-2\dot{\Omega}'-\dot{\omega}+\dot{\Omega})^2}$$

$$-\frac{5}{4}\sin i\left(1-2\cos i-3\cos^2 i\right)\sin i'\left(1-2\cos i'-3\cos^2 i'\right)\frac{\sin(\lambda'+\Omega'-\omega-2\Omega)}{\left(\dot{\lambda}'+\dot{\Omega}'-\dot{\omega}-2\dot{\Omega}\right)^2}$$

$$+\frac{5}{4}\sin i\left(1+2\cos i-3\cos^2 i\right)\sin i'\left(1+2\cos i'-3\cos^2 i'\right)\frac{\sin(\lambda'-3\Omega'-\omega+2\Omega)}{\left(\dot{\lambda}'-3\dot{\Omega}'-\dot{\omega}+2\dot{\Omega}\right)^2}$$

$$-\frac{45}{4}\sin^2 i\cos^2\frac{i}{2}\sin^2 i'\cos^2\frac{i'}{2}\frac{\sin(\lambda'+2\Omega'-\omega-3\Omega)}{\left(\dot{\lambda}'+2\dot{\Omega}'-\dot{\omega}-3\dot{\Omega}\right)^2}$$

$$+\frac{45}{4}\sin^2 i\sin^2\frac{i}{2}\sin^2 i'\sin^2\frac{i'}{2}\frac{\sin(\lambda'-4\Omega'-\omega+3\Omega)}{\left(\dot{\lambda}'-4\dot{\Omega}'-\dot{\omega}+3\dot{\Omega}\right)^2}$$

$$-\frac{5}{4}\cos^2\frac{i}{2}\left(1+10\cos i-15\cos^2 i\right)\sin^2 i'\sin^2\frac{i'}{2}\frac{\sin(3\lambda'-4\Omega'+\omega+\Omega)}{\left(3\dot{\lambda}'-4\dot{\Omega}'+\dot{\omega}+\dot{\Omega}\right)^2}$$

$$+\frac{5}{4}\sin^2\frac{i}{2}\left(1-10\cos i-15\cos^2 i\right)\sin^2 i'\cos^2\frac{i'}{2}\frac{\sin(3\lambda'-2\Omega'+\omega-\Omega)}{\left(3\dot{\lambda}'-2\dot{\Omega}'+\dot{\omega}-\dot{\Omega}\right)^2}$$

$$-5\sin i\left(1-2\cos i-3\cos^2 i\right)\sin i'\sin^4\frac{i'}{2}\frac{\sin(3\lambda'-5\Omega'+\omega+2\Omega)}{\left(3\dot{\lambda}'-5\dot{\Omega}'+\dot{\omega}+2\dot{\Omega}\right)^2}$$

$$+5\sin i\left(1+2\cos i-3\cos^2 i\right)\sin i'\cos^4\frac{i'}{2}\frac{\sin(3\lambda'-\Omega'+\omega-2\Omega)}{\left(3\dot{\lambda}'-\dot{\Omega}'+\dot{\omega}-2\dot{\Omega}\right)^2}$$

$$+15\sin^2 i\cos^2\frac{i}{2}\sin^6\frac{i'}{2}\frac{\sin(3\lambda'-6\Omega'+\omega+3\Omega)}{\left(3\dot{\lambda}'-6\dot{\Omega}'+\dot{\omega}+3\dot{\Omega}\right)^2}$$

$$-15\sin^2 i\sin^2\frac{i}{2}\cos^6\frac{i'}{2}\frac{\sin(3\lambda'+\omega-3\Omega)}{\left(3\dot{\lambda}'+\dot{\omega}-3\dot{\Omega}\right)^2}$$

$$+\frac{5}{4}\cos^2\frac{i}{2}\left(1+10\cos i-15\cos^2 i\right)\sin^2 i'\cos^2\frac{i'}{2}\frac{\sin(3\lambda'-2\Omega'-\omega-\Omega)}{\left(3\dot{\lambda}'-2\dot{\Omega}'-\dot{\omega}-\dot{\Omega}\right)^2}$$

$$-\frac{5}{4}\sin^2\frac{i}{2}\left(1-10\cos i-15\cos^2 i\right)\sin^2 i'\sin^2\frac{i'}{2}\frac{\sin(3\lambda'-4\Omega'-\omega+\Omega)}{\left(3\dot{\lambda}'-4\dot{\Omega}'-\dot{\omega}+\dot{\Omega}\right)^2}$$

$$- 5 \sin i \left(1 - 2\cos i - 3\cos^2 i\right) \sin i' \cos^4 \frac{i'}{2} \frac{\sin(3\lambda' - \Omega' - \omega - 2\Omega)}{(3\dot{\lambda}' - \dot{\Omega}' - \dot{\omega} - 2\dot{\Omega})^2}$$

$$+ 5 \sin i \left(1 + 2\cos i - 3\cos^2 i\right) \sin i' \sin^4 \frac{i'}{2} \frac{\sin(3\lambda' - 5\Omega' - \omega + 2\Omega)}{(3\dot{\lambda}' - 5\dot{\Omega}' - \dot{\omega} + 2\dot{\Omega})^2}$$

$$- 15 \sin^2 i \cos^2 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\sin(3\lambda' - \omega - 3\Omega)}{(3\dot{\lambda}' - \dot{\omega} - 3\dot{\Omega})^2}$$

$$\left. + 15 \sin^2 i \sin^2 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\sin(3\lambda' - 6\Omega' - \omega + 3\Omega)}{(3\dot{\lambda}' - 6\dot{\Omega}' - \dot{\omega} + 3\dot{\Omega})^2} \right]$$

$$+ \left(\frac{m'}{n}\right)\left(\frac{a}{a'^4}\right) \frac{1}{\sin i \sqrt{1 - e^2}} \cdot \frac{175}{64} e^3 \left[- \frac{3}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 \frac{i'}{2} \left(1 - 10\cos i' - 15\cos^2 i'\right) \frac{\sin(\lambda' - 2\Omega' + 3\omega + \Omega)}{(\dot{\lambda}' - 2\dot{\Omega}' + 3\dot{\omega} + \dot{\Omega})^2} \right.$$

$$+ \frac{3}{16} \sin^2 i \sin^2 \frac{i}{2} \cos^2 \frac{i'}{2} \left(1 + 10\cos i' - 15\cos^2 i'\right) \frac{\sin(\lambda' + 3\omega - \Omega)}{(\dot{\lambda}' + 3\dot{\omega} - \dot{\Omega})^2}$$

$$- \frac{3}{4} \sin i \cos^4 \frac{i}{2} \sin i' \left(1 + 2\cos i' - 3\cos^2 i'\right) \frac{\sin(\lambda' - 3\Omega' + 3\omega + 2\Omega)}{(\dot{\lambda}' - 3\dot{\Omega}' + 3\dot{\omega} + 2\dot{\Omega})^2}$$

$$+ \frac{3}{4} \sin i \sin^4 \frac{i}{2} \sin i' \left(1 - 2\cos i' - 3\cos^2 i'\right) \frac{\sin(\lambda' + \Omega' + 3\omega - 2\Omega)}{(\dot{\lambda}' + \dot{\Omega}' + 3\dot{\omega} - 2\dot{\Omega})^2}$$

$$+ \frac{9}{4} \cos^6 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(\lambda' - 4\Omega' + 3\omega + 3\Omega)}{(\dot{\lambda}' - 4\dot{\Omega}' + 3\dot{\omega} + 3\dot{\Omega})^2}$$

$$- \frac{9}{4} \sin^6 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(\lambda' + 2\Omega' + 3\omega - 3\Omega)}{(\dot{\lambda}' + 2\dot{\Omega}' + 3\dot{\omega} - 3\dot{\Omega})^2}$$

$$+ \frac{3}{16} \sin^2 i \cos^2 \frac{i}{2} \cos^2 \frac{i'}{2} \left(1 + 10\cos i' - 15\cos^2 i'\right) \frac{\sin(\lambda' - 3\omega - \Omega)}{(\dot{\lambda}' - 3\dot{\omega} - \dot{\Omega})^2}$$

$$- \frac{3}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 \frac{i'}{2} \left(1 - 10\cos i' - 15\cos^2 i'\right) \frac{\sin(\lambda' - 2\Omega' - 3\omega + \Omega)}{(\dot{\lambda}' - 2\dot{\Omega}' - 3\dot{\omega} + \dot{\Omega})^2}$$

$$- \frac{3}{4} \sin i \cos^4 \frac{i}{2} \sin i' \left(1 - 2\cos i' - 3\cos^2 i'\right) \frac{\sin(\lambda' + \Omega' - 3\omega - 2\Omega)}{(\dot{\lambda}' + \dot{\Omega}' - 3\dot{\omega} - 2\dot{\Omega})^2}$$

$$+ \frac{3}{4} \sin i \sin^4 \frac{i}{2} \sin i' \left(1 + 2\cos i' - 3\cos^2 i'\right) \frac{\sin(\lambda' - 3\Omega' - 3\omega + 2\Omega)}{(\dot{\lambda}' - 3\dot{\Omega}' - 3\dot{\omega} + 2\dot{\Omega})^2}$$

$$- \frac{9}{4} \cos^6 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(\lambda' + 2\Omega' - 3\omega - 3\Omega)}{(\dot{\lambda}' + 2\dot{\Omega}' - 3\dot{\omega} - 3\dot{\Omega})^2}$$

$$+ \frac{9}{4} \sin^6 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(\lambda' - 4\Omega' - 3\omega + 3\Omega)}{(\dot{\lambda}' - 4\dot{\Omega}' - 3\dot{\omega} + 3\dot{\Omega})^2}$$

$$+ \frac{15}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(3\lambda' - 4\Omega' + 3\omega + \Omega)}{(3\dot{\lambda}' - 4\dot{\Omega}' + 3\dot{\omega} + \dot{\Omega})^2}$$

$$- \frac{15}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(3\lambda' - 2\Omega' + 3\omega - \Omega)}{(3\dot{\lambda}' - 2\dot{\Omega}' + 3\dot{\omega} - \dot{\Omega})^2}$$

$$- 3 \sin i \cos^4 \frac{i}{2} \sin i' \sin^4 \frac{i'}{2} \frac{\sin(3\lambda' - 5\Omega' + 3\omega + 2\Omega)}{(3\dot{\lambda}' - 5\dot{\Omega}' + 3\dot{\omega} + 2\dot{\Omega})^2}$$

$$+ 3 \sin i \sin^4 \frac{i}{2} \sin i' \cos^4 \frac{i'}{2} \frac{\sin(3\lambda' - \Omega' + 3\omega - 2\Omega)}{(3\dot{\lambda}' - \dot{\Omega}' + 3\dot{\omega} - 2\dot{\Omega})^2}$$

$$+ 3 \cos^6 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\sin(3\lambda' - 6\Omega' + 3\omega + 3\Omega)}{(3\dot{\lambda}' - 6\dot{\Omega}' + 3\dot{\omega} + 3\dot{\Omega})^2}$$

$$- 3 \sin^6 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\sin(3\lambda' + 3\omega - 3\Omega)}{(3\dot{\lambda}' + 3\dot{\omega} - 3\dot{\Omega})^2}$$

$$- \frac{15}{16} \sin^2 i \cos^2 \frac{i}{2} \sin^2 i' \cos^2 \frac{i'}{2} \frac{\sin(3\lambda' - 2\Omega' - 3\omega - \Omega)}{(3\dot{\lambda}' - 2\dot{\Omega}' - 3\dot{\omega} - \dot{\Omega})^2}$$

$$+ \frac{15}{16} \sin^2 i \sin^2 \frac{i}{2} \sin^2 i' \sin^2 \frac{i'}{2} \frac{\sin(3\lambda' - 4\Omega' - 3\omega + \Omega)}{(3\dot{\lambda}' - 4\dot{\Omega}' - 3\dot{\omega} + \dot{\Omega})^2}$$

$$- 3 \sin i \cos^4 \frac{i}{2} \sin i' \cos^4 \frac{i'}{2} \frac{\sin(3\lambda' - \Omega' - 3\omega - 2\Omega)}{(3\dot{\lambda}' - \dot{\Omega}' - 3\dot{\omega} - 2\dot{\Omega})^2}$$

$$+ 3 \sin i \sin^4 \frac{i}{2} \sin i' \sin^4 \frac{i'}{2} \frac{\sin(3\lambda' - 5\Omega' - 3\omega + 2\Omega)}{(3\dot{\lambda}' - 5\dot{\Omega}' - 3\dot{\omega} + 2\dot{\Omega})^2}$$

$$- 3 \cos^6 \frac{i}{2} \cos^6 \frac{i'}{2} \frac{\sin(3\lambda' - 3\omega - 3\Omega)}{(3\dot{\lambda}' - 3\dot{\omega} - 3\dot{\Omega})^2}$$

$$\left. + 3 \sin^6 \frac{i}{2} \sin^6 \frac{i'}{2} \frac{\sin(3\lambda' - 6\Omega' - 3\omega + 3\Omega)}{(3\dot{\lambda}' - 6\dot{\Omega}' - 3\dot{\omega} + 3\dot{\Omega})^2} \right] .$$

Also,

$$\frac{d\dot{\ell}}{de} = n \left\{ - \frac{9J_2a_e^2e}{4a^2(1-e^2)^{5/2}} (1 - 3\cos^2 i) + \frac{3J_2^2a_e^4e}{128\,a^4(1-e^2)^{9/2}} [20 - 125e^2 + 96\sqrt{1-e^2} \right.$$

$$- 6(40 - 75e^2 + 96\sqrt{1-e^2})\cos^2 i + (860 - 125e^2 + 864\sqrt{1-e^2})\cos^4 i]$$

$$\left. - \frac{45J_4a_e^4e(2+5e^2)}{128a^4(1-e^2)^{9/2}} (3 - 30\cos^2 i + 35\cos^4 i) \right\} ;$$

$$\frac{d\dot{\ell}}{di} = n \left\{ - \frac{9J_2a_e^2 \sin i \cos i}{2a^2(1-e^2)^{3/2}} + \frac{3J_2^2a_e^4 \sin i \cos i}{32a^4(1-e^2)^{7/2}} [3(10 - 15e^2 + 16\sqrt{1-e^2}) \right.$$

$$\left. - (130 - 25e^2 + 144\sqrt{1-e^2})\cos^2 i] - \frac{225J_4a_e^4e^2 \sin i \cos i}{32a^4(1-e^2)^{7/2}} (3 - 7\cos^2 i) \right\} ;$$

$$\frac{d\dot{\omega}}{de} = n \left\{ - \frac{3J_2a_e^2e}{a^2(1-e^2)^3} (1 - 5\cos^2 i) + \frac{3J_2^2a_e^4e}{64a^4(1-e^2)^5} [-64 - 75e^2 + 84\sqrt{1-e^2} \right.$$

$$- 6(3 - 63e^2 + 112\sqrt{1-e^2})\cos^2 i + 5(335 - 27e^2 + 252\sqrt{1-e^2})\cos^4 i]$$

$$\left. - \frac{15J_4a_e^4e}{64a^4(1-e^2)^5} [3(19 - 9e^2) - 54(13 + 7e^2)\cos^2 i + 7(139 + 81e^2)\cos^4 i] \right\} ;$$

$$\frac{d\dot{\omega}}{di} = n \left\{ - \frac{15J_2a_e^2 \sin i \cos i}{2a^2(1-e^2)^2} + \frac{3J_2^2a_e^4 \sin i \cos i}{32a^4(1-e^2)^4} [3(6 - 21e^2 + 32\sqrt{1-e^2}) \right.$$

$$\left. - 5(86 - 9e^2 + 72\sqrt{1-e^2})\cos^2 i] - \frac{15J_4a_e^4 \sin i \cos i}{32a^4(1-e^2)^4} [9(8 + 7e^2) - 7(28 + 27e^2)\cos^2 i] \right\} ;$$

$$\frac{d\dot{\Omega}}{de} = n \left\{ - \frac{6J_2a_e^2e \cos i}{a^2(1-e^2)^3} + \frac{3J_2^2a_e^4e \cos i}{16a^4(1-e^2)^5} [7 - 27e^2 + 42\sqrt{1-e^2} \right.$$

$$\left. - (155 - 15e^2 + 126\sqrt{1-e^2})\cos^2 i] - \frac{15J_4a_e^4e(11 + 9e^2)\cos i}{16a^4(1-e^2)^5} (3 - 7\cos^2 i) \right\} ;$$

and

$$\frac{d\dot{\Omega}}{di} = n \left\{ \frac{3J_2a_e^2 \sin i}{2a^2(1-e^2)^2} - \frac{3J_2^2a_e^4 \sin i}{32a^4(1-e^2)^4} [4 - 9e^2 + 12\sqrt{1-e^2} \right.$$

$$\left. - 3(40 - 5e^2 + 36\sqrt{1-e^2})\cos^2 i] + \frac{45J_4a_e^4(2 + 3e^2)\sin i}{32a^4(1-e^2)^4} (1 - 7\cos^2 i) \right\} .$$

SOLAR RADIATION PRESSURE

Referring to either Reference 3 or Reference 4, the long-period part of the solar radiation pressure disturbing function is

$$R_p = -F\frac{3ae}{2}\left[\cos^2\frac{i}{2}\sin^2\frac{i_\odot}{2}\cos(\omega+\Omega+\lambda_\odot) + \cos^2\frac{i}{2}\cos^2\frac{i_\odot}{2}\cos(\omega+\Omega-\lambda_\odot) + \sin^2\frac{i}{2}\cos^2\frac{i_\odot}{2}\cos(\omega-\Omega+\lambda_\odot)\right.$$

$$\left. + \sin^2\frac{i}{2}\sin^2\frac{i_\odot}{2}\cos(\omega-\Omega-\lambda_\odot) - \frac{1}{2}\sin i\,\sin i_\odot\cos(\omega+\lambda_\odot) + \frac{1}{2}\sin i\,\sin i_\odot\cos(\omega-\lambda_\odot)\right]$$

where

$$F = \text{Force constant} = (-4.63\times 10^{-5}\ \text{dyn/cm}^2)\times(A/m)\ .$$

The perturbations in the orbital elements caused by solar radiation pressure are derived in the same manner as those due to lunar and solar gravitation. If we replace the series A_i in Equations 4 and 5 by the B_i defined below, we will have the expressions for the long-period solar radiation pressure effects. The various B_i are as follows:

$$B_1 = \int\frac{\partial R_p}{\partial\omega}dt$$

$$= -F\frac{3ae}{2}\left[\cos^2\frac{i}{2}\sin^2\frac{i_\odot}{2}\frac{\cos(\omega+\Omega+\lambda_\odot)}{\dot\omega+\dot\Omega+\dot\lambda_\odot} + \cos^2\frac{i}{2}\cos^2\frac{i_\odot}{2}\frac{\cos(\omega+\Omega-\lambda_\odot)}{\dot\omega+\dot\Omega-\dot\lambda_\odot}\right.$$

$$+ \sin^2\frac{i}{2}\cos^2\frac{i_\odot}{2}\frac{\cos(\omega-\Omega+\lambda_\odot)}{\dot\omega-\dot\Omega+\dot\lambda_\odot} + \sin^2\frac{i}{2}\sin^2\frac{i_\odot}{2}\frac{\cos(\omega-\Omega-\lambda_\odot)}{\dot\omega-\dot\Omega-\dot\lambda_\odot}$$

$$\left. -\frac{1}{2}\sin i\,\sin i_\odot\frac{\cos(\omega+\lambda_\odot)}{\dot\omega+\dot\lambda_\odot} + \frac{1}{2}\sin i\,\sin i_\odot\frac{\cos(\omega-\lambda_\odot)}{\dot\omega-\dot\lambda_\odot}\right]\ ;$$

$$B_2 = \int\frac{\partial R_p}{\partial\Omega}dt$$

$$= -F\frac{3ae}{2}\left[\cos^2\frac{i}{2}\sin^2\frac{i_\odot}{2}\frac{\cos(\omega+\Omega+\lambda_\odot)}{\dot\omega+\dot\Omega+\dot\lambda_\odot} + \cos^2\frac{i}{2}\cos^2\frac{i_\odot}{2}\frac{\cos(\omega+\Omega-\lambda_\odot)}{\dot\omega+\dot\Omega-\dot\lambda_\odot}\right.$$

$$\left. - \sin^2\frac{i}{2}\cos^2\frac{i_\odot}{2}\frac{\cos(\omega-\Omega+\lambda_\odot)}{\dot\omega-\dot\Omega+\dot\lambda_\odot} - \sin^2\frac{i}{2}\sin^2\frac{i_\odot}{2}\frac{\cos(\omega-\Omega-\lambda_\odot)}{\dot\omega-\dot\Omega-\dot\lambda_\odot}\right]\ ;$$

$$B_3 = \int \frac{\partial R_p}{\partial a} dt$$

$$= -F\frac{3e}{2}\left[\cos^2\frac{i}{2}\sin^2\frac{i_\odot}{2}\frac{\sin(\omega+\Omega+\lambda_\odot)}{\dot\omega+\dot\Omega+\dot\lambda_\odot} + \cos^2\frac{i}{2}\cos^2\frac{i_\odot}{2}\frac{\sin(\omega+\Omega-\lambda_\odot)}{\dot\omega+\dot\Omega-\dot\lambda_\odot}\right.$$

$$+ \sin^2\frac{i}{2}\cos^2\frac{i_\odot}{2}\frac{\sin(\omega-\Omega+\lambda_\odot)}{\dot\omega-\dot\Omega+\dot\lambda_\odot} + \sin^2\frac{i}{2}\sin^2\frac{i_\odot}{2}\frac{\sin(\omega-\Omega-\lambda_\odot)}{\dot\omega-\dot\Omega-\dot\lambda_\odot}$$

$$\left. -\frac{1}{2}\sin i \sin i_\odot\frac{\sin(\omega+\lambda_\odot)}{\dot\omega+\dot\lambda_\odot} + \frac{1}{2}\sin i \sin i_\odot\frac{\sin(\omega-\lambda_\odot)}{\dot\omega-\dot\lambda_\odot}\right] \quad ;$$

$$B_4 = \int \frac{\partial R_p}{\partial e} dt$$

$$= \frac{a}{e} B_3 \quad ;$$

$$B_5 = \int \frac{\partial R_p}{\partial i} dt$$

$$= F\frac{3ae}{4}\left[\sin i \sin^2\frac{i_\odot}{2}\frac{\sin(\omega+\Omega+\lambda_\odot)}{\dot\omega+\dot\Omega+\dot\lambda_\odot} + \sin i \cos^2\frac{i_\odot}{2}\frac{\sin(\omega+\Omega-\lambda_\odot)}{\dot\omega+\dot\Omega-\dot\lambda_\odot}\right.$$

$$- \sin i \cos^2\frac{i_\odot}{2}\frac{\sin(\omega-\Omega+\lambda_\odot)}{\dot\omega-\dot\Omega+\dot\lambda_\odot} - \sin i \sin^2\frac{i_\odot}{2}\frac{\sin(\omega-\Omega-\lambda_\odot)}{\dot\omega-\dot\Omega-\dot\lambda_\odot}$$

$$\left. + \cos i \sin i_\odot\frac{\sin(\omega+\lambda_\odot)}{\dot\omega+\dot\lambda_\odot} - \cos i \sin i_\odot\frac{\sin(\omega-\lambda_\odot)}{\dot\omega-\dot\lambda_\odot}\right] \quad ;$$

$$B_6 = \int \delta e \; dt$$

$$= -\frac{\sqrt{1-e^2}}{n a^2 e}\int B_1 \; dt$$

$$= \frac{3}{2}F\frac{\sqrt{1-e^2}}{na}\left[\cos^2\frac{i}{2}\sin^2\frac{i_\odot}{2}\frac{\sin(\omega+\Omega+\lambda_\odot)}{(\dot\omega+\dot\Omega+\dot\lambda_\odot)^2} + \cos^2\frac{i}{2}\cos^2\frac{i_\odot}{2}\frac{\sin(\omega+\Omega-\lambda_\odot)}{(\dot\omega+\dot\Omega-\dot\lambda_\odot)^2}\right.$$

$$+ \sin^2\frac{i}{2}\cos^2\frac{i_\odot}{2}\frac{\sin(\omega-\Omega+\lambda_\odot)}{(\dot\omega-\dot\Omega+\dot\lambda_\odot)^2} + \sin^2\frac{i}{2}\sin^2\frac{i_\odot}{2}\frac{\sin(\omega-\Omega-\lambda_\odot)}{(\dot\omega-\dot\Omega-\dot\lambda_\odot)^2}$$

$$\left. -\frac{1}{2}\sin i \sin i_\odot\frac{\sin(\omega+\lambda_\odot)}{(\dot\omega+\dot\lambda_\odot)^2} + \frac{1}{2}\sin i \sin i_\odot\frac{\sin(\omega-\lambda_\odot)}{(\dot\omega-\dot\lambda_\odot)^2}\right] \quad ;$$

and

$$B_7 = \int \delta i \, dt$$

$$= \frac{\cos i}{n a^2 \sqrt{1-e^2} \sin i} \int B_1 \, dt - \frac{1}{n a^2 \sqrt{1-e^2} \sin i} \int B_2 \, dt$$

$$= -\frac{e \cot i}{1-e^2} B_6$$

$$+ \frac{3}{2} F \frac{e}{n a \sqrt{1-e^2} \sin i} \left[\cos^2 \frac{i}{2} \sin^2 \frac{i_\odot}{2} \frac{\sin(\omega + \Omega + \lambda_\odot)}{(\dot\omega + \dot\Omega + \dot\lambda_\odot)^2} + \cos^2 \frac{i}{2} \cos^2 \frac{i_\odot}{2} \frac{\sin(\omega + \Omega - \lambda_\odot)}{(\dot\omega + \dot\Omega - \dot\lambda_\odot)^2} \right.$$

$$\left. - \sin^2 \frac{i}{2} \cos^2 \frac{i_\odot}{2} \frac{\sin(\omega - \Omega + \lambda_\odot)}{(\dot\omega - \dot\Omega + \dot\lambda_\odot)^2} - \sin^2 \frac{i}{2} \sin^2 \frac{i_\odot}{2} \frac{\sin(\omega - \Omega - \lambda_\odot)}{(\dot\omega - \dot\Omega - \dot\lambda_\odot)^2} \right] .$$

SOLAR AND LUNAR QUANTITIES

The various quantities (i', λ', etc.) needed in the perturbation formulas can be obtained from References 6 and 7. However, the formulas needed to compute these quantities are reproduced here for convenience. In addition, the transformations necessary to refer the lunar quantities to the earth's equatorial plane are given. So that the reader need not refer to Reference 6 or 7, the following information is given:

For January 1.0, 1960, the number of days since January 0.5, 1900, is 21914.5. To find T for a later epoch, add to this figure the number of days between January 1.0, 1960, and the epoch, and divide by 36525.

For the sun:

$m_\odot = 0.999997$

$e_\odot = 0.01675104 - 0.00004180T - 0.00000013T^2$

$n_\odot = 0\overset{\circ}{.}98560027/\text{day}$

$\lambda_\odot = 279\overset{\circ}{.}69668 + 36000\overset{\circ}{.}76892T + 0\overset{\circ}{.}00030T^2$

$\omega_\odot = 281\overset{\circ}{.}22083 + 1\overset{\circ}{.}71918T + 0\overset{\circ}{.}00045T^2$

$\Omega_\odot = 0$

$i_\odot = 23\overset{\circ}{.}452294 - 0\overset{\circ}{.}013013T - 0\overset{\circ}{.}000002T^2.$

The mean motions of the angular variables are determined by differentiating the proper equations above with respect to T.

For the moon, the basic quantities (referred to the ecliptic) are as follows:

$m_{\mathbb{C}} = 0.012150668$

$e_{\mathbb{C}} = 0.054900489$

$\mathbb{C} = 270^\circ\!.43416 + 481267^\circ\!.88314T - 0^\circ\!.00113T^2$

$\Gamma'' = 334^\circ\!.32956 + 4069^\circ\!.03403T - 0^\circ\!.01033T^2 - 0^\circ\!.00001T^3$

$\Omega'' = 259^\circ\!.18328 - 1934^\circ\!.14201T + 0^\circ\!.00208T^2$

$n_{\mathbb{C}} = 13^\circ\!.064993/\text{day}$

$i'' = 5^\circ\!.1453964.$

(Note: Γ'' and Ω'' are written Γ' and Ω in References 6 and 7.)

The transformations needed to refer the lunar quantities to the basic reference plane may be derived through the use of Figure 1. Thus,

$$\cos i_{\mathbb{C}} = \cos i'' \cos i_{\odot} - \sin i'' \sin i_{\odot} \cos \Omega'' ,$$

$$\sin \Omega_{\mathbb{C}} = \frac{\sin i'' \sin \Omega''}{\sin i_{\mathbb{C}}} ,$$

$$\cos \Omega_{\mathbb{C}} = \frac{\cos i'' - \cos i_{\odot} \cos i_{\mathbb{C}}}{\sin i_{\odot} \sin i_{\mathbb{C}}} ,$$

$$\sin \left[\omega_{\mathbb{C}} - (\Gamma'' - \Omega'')\right] = \frac{\sin i_{\odot} \sin \Omega''}{\sin i_{\mathbb{C}}} ,$$

$$\cos \left[\omega_{\mathbb{C}} - (\Gamma'' - \Omega'')\right] = \frac{\cos i_{\odot} - \cos i'' \cos i_{\mathbb{C}}}{\sin i'' \sin i_{\mathbb{C}}} ,$$

and

$$\lambda_{\mathbb{C}} = \Omega_{\mathbb{C}} + \omega_{\mathbb{C}} + (\mathbb{C} - \Gamma'') .$$

Then,

$$\frac{di_{\mathbb{C}}}{dt} = \frac{\frac{di_{\odot}}{dt}\left(\cos i'' \sin i_{\odot} + \sin i'' \cos i_{\odot} \cos \Omega''\right) - \dot{\Omega}'' \sin i'' \sin i_{\odot} \sin \Omega''}{\sin i_{\mathbb{C}}} ,$$

$$\dot{\Omega}_{\mathbb{C}} = \frac{\sin i''}{\sin i_{\mathbb{C}} \cos \Omega_{\mathbb{C}}} \left(\dot{\Omega}'' \cos \Omega'' - \frac{\cos i_{\mathbb{C}} \sin \Omega''}{\sin i_{\mathbb{C}}} \frac{di_{\mathbb{C}}}{dt}\right) ,$$

$$\dot{\omega}_{\mathbb{C}} = \dot{\Gamma}'' - \dot{\Omega}'' + \frac{1}{\sin i_{\mathbb{C}} \cos\left[\omega_{\mathbb{C}} - (\Gamma'' - \Omega'')\right]} \left(\frac{di_{\odot}}{dt} \cos i_{\odot} \sin\Omega'' + \dot{\Omega}'' \sin i_{\odot} \cos\Omega'' - \frac{\sin i_{\odot} \cos i_{\mathbb{C}} \sin\Omega''}{\sin i_{\mathbb{C}}} \frac{di_{\mathbb{C}}}{dt} \right),$$

and

$$\dot{\lambda}_{\mathbb{C}} = \dot{\Omega}_{\mathbb{C}} + \dot{\omega}_{\mathbb{C}} + (\dot{\mathbb{C}} - \dot{\Gamma}'') .$$

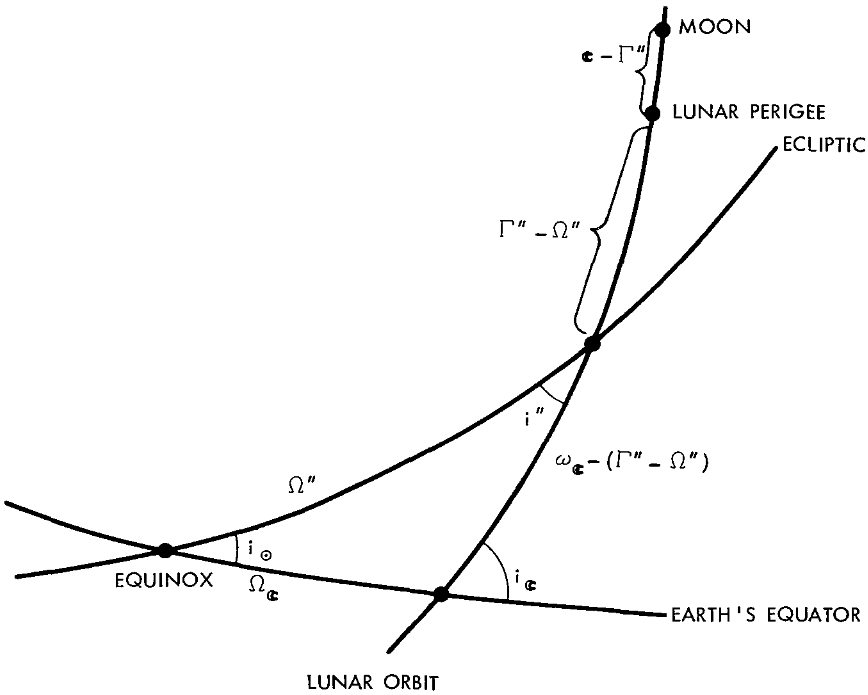

Figure 1—Geometrical relations between angular elements.

COMPARISON WITH OBLATENESS

One can obtain an estimate of the magnitude of the secular and long period luni-solar perturbations relative to those due to the earth's oblateness by a study of the parameters appearing as factors in the disturbing functions.

It can be seen from the 1959 paper of Brouwer (Reference 5) that the long-period part of the Hamiltonian F^*_{2p} appears with the small factors J_2^2/a^5, J_3/a^4, J_4/a^5, and J_5/a^6, and the secular parts F_{1s} and F^*_{2s} have J_2/a^3, J_2^2/a^5, and J_4/a^5 as small factors (F^*_{2p}, although second order, gives rise to first-order long-period perturbations). The secular and long-period gravitational disturbing functions of the sun and moon have the factors $m_{\odot} n_{\odot}^2 a^2$ and $m_{\mathbb{C}} n_{\mathbb{C}}^2 a^2$, respectively. The long-period solar radiation pressure disturbing function appears with $-Fa$ as a multiplier (for illustration, the value of A/m for Telstar 2, which is about 0.074860427 cm ^{2}/gm, was chosen for use in this analysis).

Table 1 in Appendix B gives a listing of these parameters for increasing values of a; here, the values of the zonal harmonic coefficients used were those adopted by the Goddard Computing Center (see Appendix A). In addition, the radius of the earth has been chosen as the unit of distance, time is expressed in the Vanguard unit (806.832 sec.), and angular measure is in radians.

Assuming that divisors of approximately the same order arise in the integration of both the oblateness and luni-solar disturbing functions, we can see that at about $a \approx 1.2$ the long-period effects due to luni-solar gravitation are about the same as those arising from J_5. Further, at $a \approx 1.9$ the luni-solar long-period effects are just as important as those due to any one of J_2^2, J_3, J_4, or J_5. Also, at $a \approx 3$, solar radiation pressure is as significant as the oblateness of the earth. For increasing values of a, of course, the luni-solar forces became progressively larger while the effects due to the earth's oblateness diminish.

As far as the secular terms are concerned, the luni-solar effects became comparable to the oblateness effects at about $a \approx 5$. Therefore, the integration becomes invalid at this point, and the formulas should not be used for satellites whose orbits have semimajor axes exceeding this value.

The next section presents a study of the effects of luni-solar forces on the motions of Relay 1 and Telstar 2.

ANALYSIS OF RELAY 1 AND TELSTAR 2

An examination of the mean orbital elements of Relay 1 and Telstar 2 reveals quite substantial long-period variations remaining in the eccentricity, inclination, argument of perigee, and longitude of ascending node. These elements are the "double-primed" variables of Brouwer (see Reference 5), so the effects due to the earth's zonal harmonics through J_5 have been accounted for. Since the two satellites are high enough so that air drag may be neglected, it was felt that these perturbations might be due to luni-solar forces. For the purpose of this analysis, only the long-period terms arising from the second Legendre polynomial are considered.

Data for Relay 1 and Telstar 2 spanning 630 and 525 days, respectively, were analyzed (see Appendix B). The mean values of eccentricity and inclination are given in Tables B2 and B5 (Appendix B) in the columns headed "e" and " i," while the " e_c" and " i_c" columns list the corrected values; i.e., if δe is the perturbation in e computed from the formulas in this paper, then

$$e_c = e - \delta e .$$

(The double-primed elements for each epoch were used to compute all luni-solar perturbations.)

Figures 2 to 5 give plots of the uncorrected and corrected values of e and i against time. It is clear that the major portions of the perturbations were indeed due to luni-solar forces.

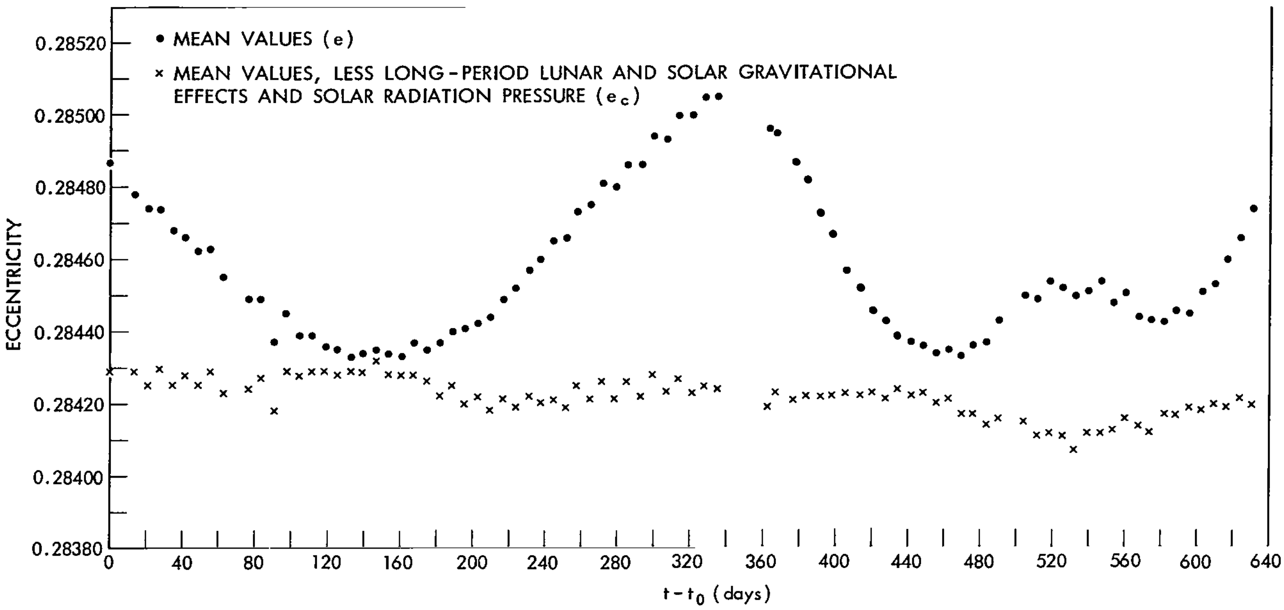

Figure 2—Eccentricity of Relay 1.

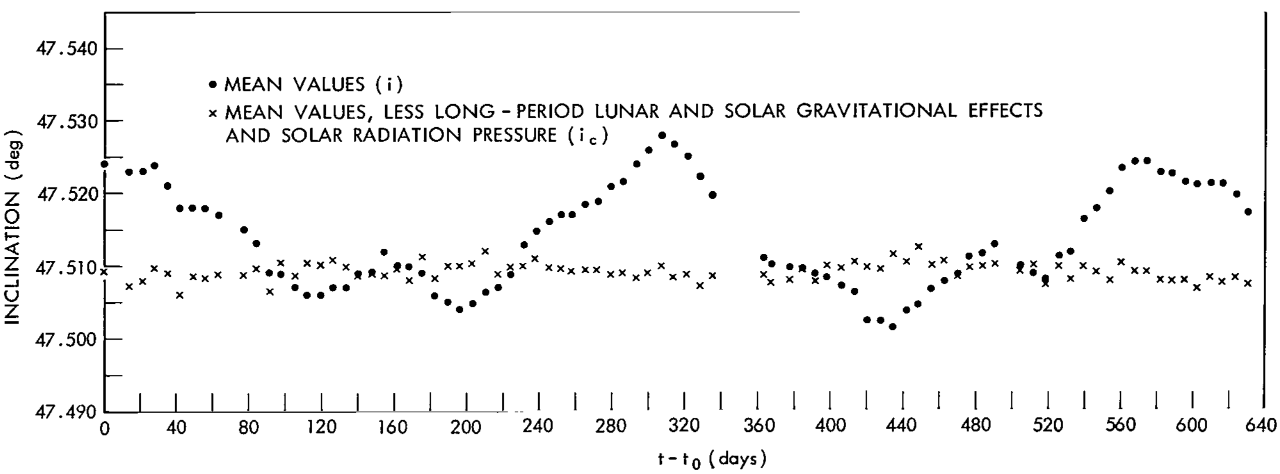

Figure 3—Inclination of Relay 1.

The argument of perigee and the longitude of ascending node were handled in a somewhat different fashion because of the presence of secular effects. First, the existence of long-period variations in the double-primed elements was detected by examining the residuals from least-squares fits to a constant plus a secular term. Then the long-period luni-solar perturbations

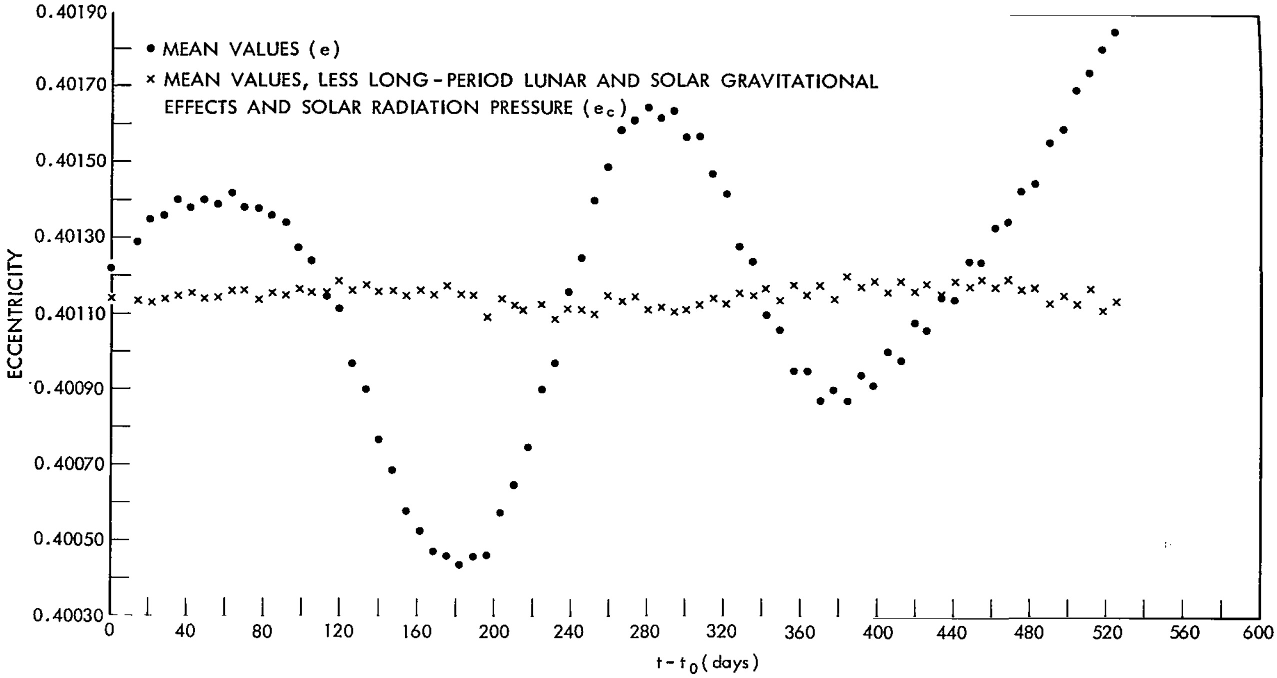

Figure 4—Eccentricity of Telstar 2.

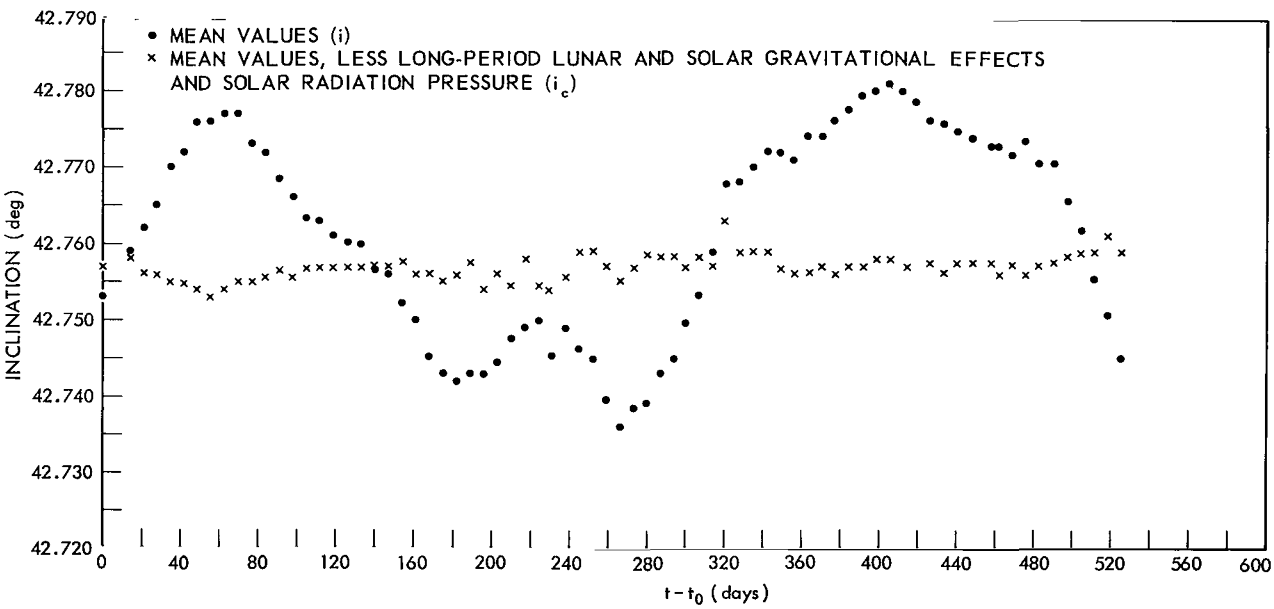

Figure 5—Inclination of Telstar 2.

were subtracted from the double-primed elements, additional least squares analyses of the same type were made, and the residuals were again examined. The least squares results are as follows:

(1) Relay 1:

$$\omega = 349^\circ\!.704 + (1^\circ\!.21264/\text{day})\,(t - t_0)$$

$$\omega_c = \omega - \delta\omega = 349^\circ\!.850 + (1^\circ\!.21206/\text{day})\,(t - t_0)$$

$$\Omega = 37^\circ\!.035 - (1^\circ\!.27953/\text{day})\,(t - t_0)$$

$$\Omega_c = \Omega - \delta\Omega = 36^\circ\!.885 - (1^\circ\!.27895/\text{day})\,(t - t_0)$$

(2) Telstar 2:

$$\omega = 297^\circ\!.183 + (1^\circ\!.21685/\text{day})\,(t - t_0)$$

$$\omega_c = \omega - \delta\omega = 297^\circ\!.242 + (1^\circ\!.21716/\text{day})\,(t - t_0)$$

$$\Omega = 76^\circ\!.503 - (1^\circ\!.05483/\text{day})\,(t - t_0)$$

$$\Omega_c = \Omega - \delta\Omega = 76^\circ\!.446 - (1^\circ\!.05497/\text{day})\,(t - t_0)$$

Tables 3, 4, 6 and 7 (Appendix B) give the uncorrected and corrected values of ω and Ω in addition to the residuals from the least squares analyses. The other entries in the tables are self-explanatory — they were computed under the assumption that the secular motions obtained from the least squares fits to the corrected data are more nearly the true values.

Figures 6 to 13 indicate that the principal remaining variations in the argument of perigee and longitude of ascending node were caused by the sun and the moon. Figures 7, 9, 11, and 13 reflect the presence of long-period luni-solar perturbations which appear to be secular effects over the time intervals studied; this explains the differences in the secular motions obtained in the least squares analyses of uncorrected and corrected data.

To illustrate the fact that long-period luni-solar effects can be as significant as first-order long-period oblateness effects, let us examine the perturbations in the eccentricity of Relay 1 and Telstar 2. Tables 8 and 9 present the amplitudes of the principal long-period terms appearing in the formulas for the luni-solar and oblateness effects, along with the corresponding arguments and periods (the formulas for the oblateness terms were obtained from Reference 5). It is clear that the luni-solar effects are just as important as oblateness perturbations. Table 1 also indicates this, since the orbits of Telstar 2 and Relay 1 have semimajor axes of about 1.9 and 1.7 earth radii, respectively.

(Manuscript received February 9,1966)

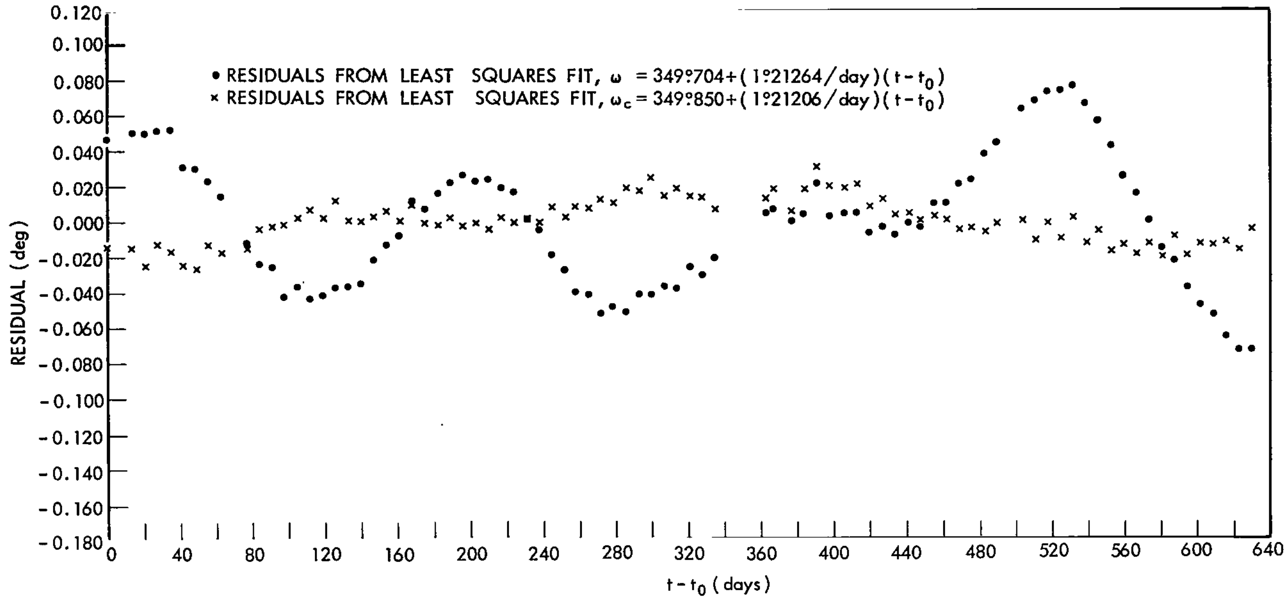

Figure 6—Residuals from least squares fits to argument of perigee for Relay 1.

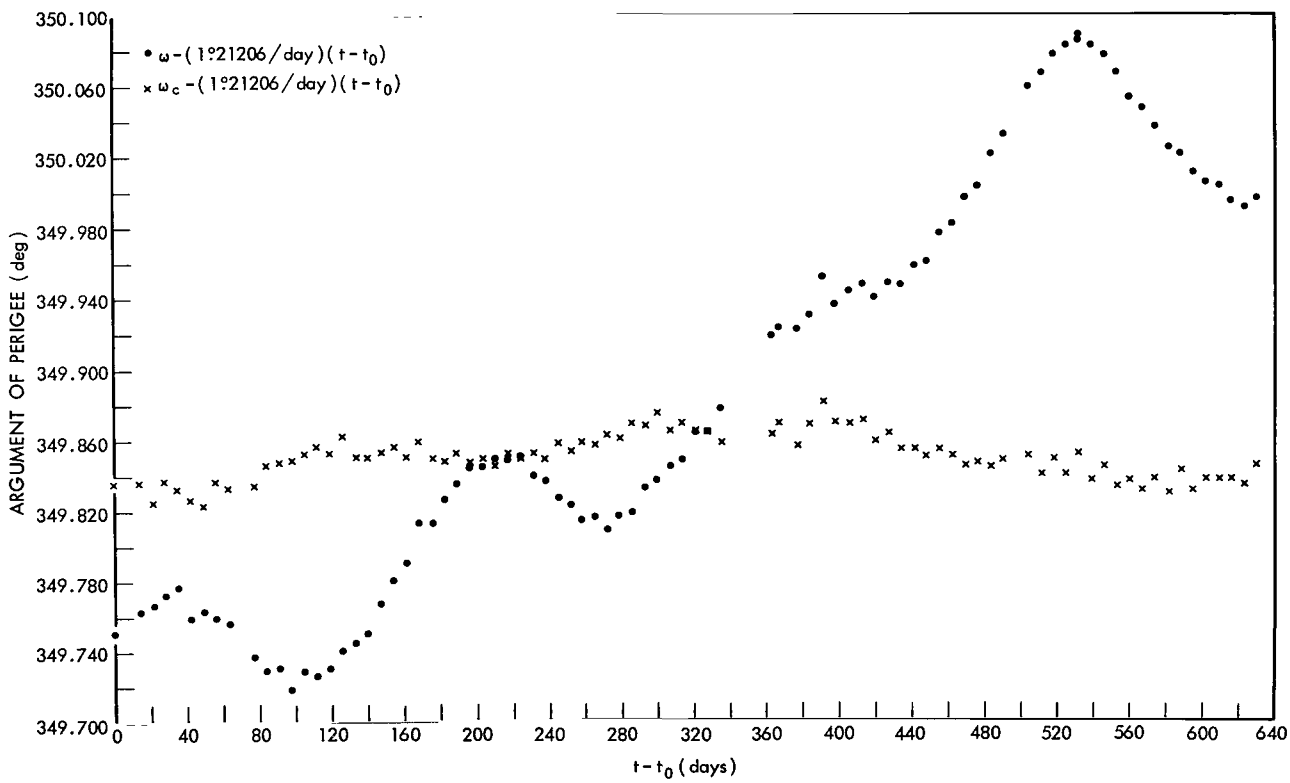

Figure 7—Argument of perigee for Relay 1.

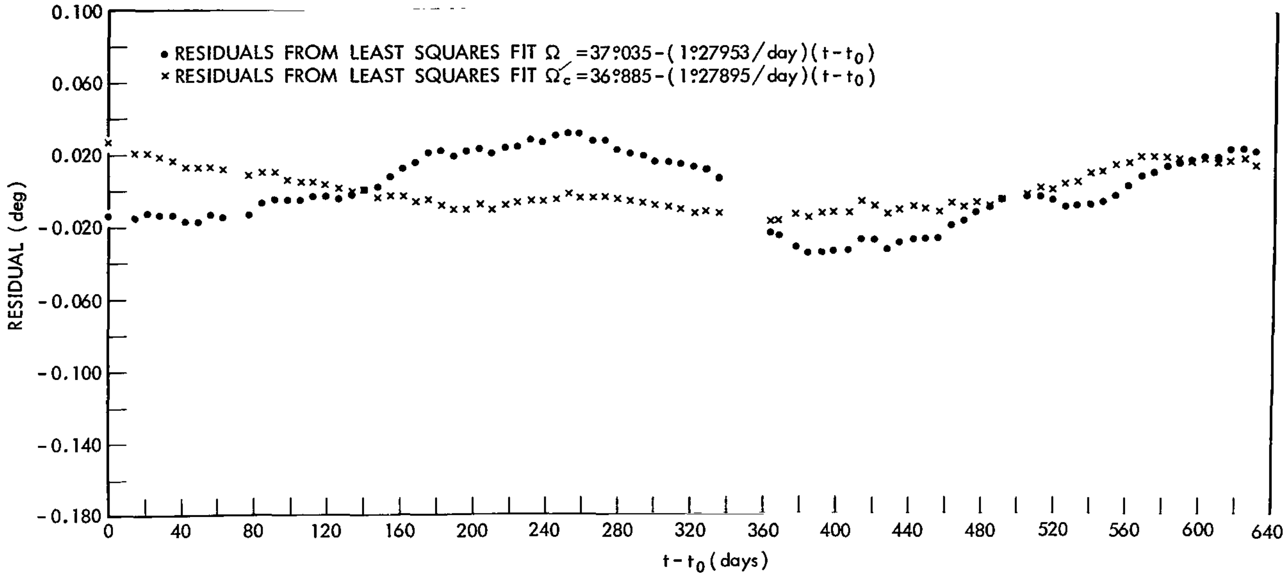

Figure 8—Residuals from least squares fits to longitude of ascending node for Relay 1.

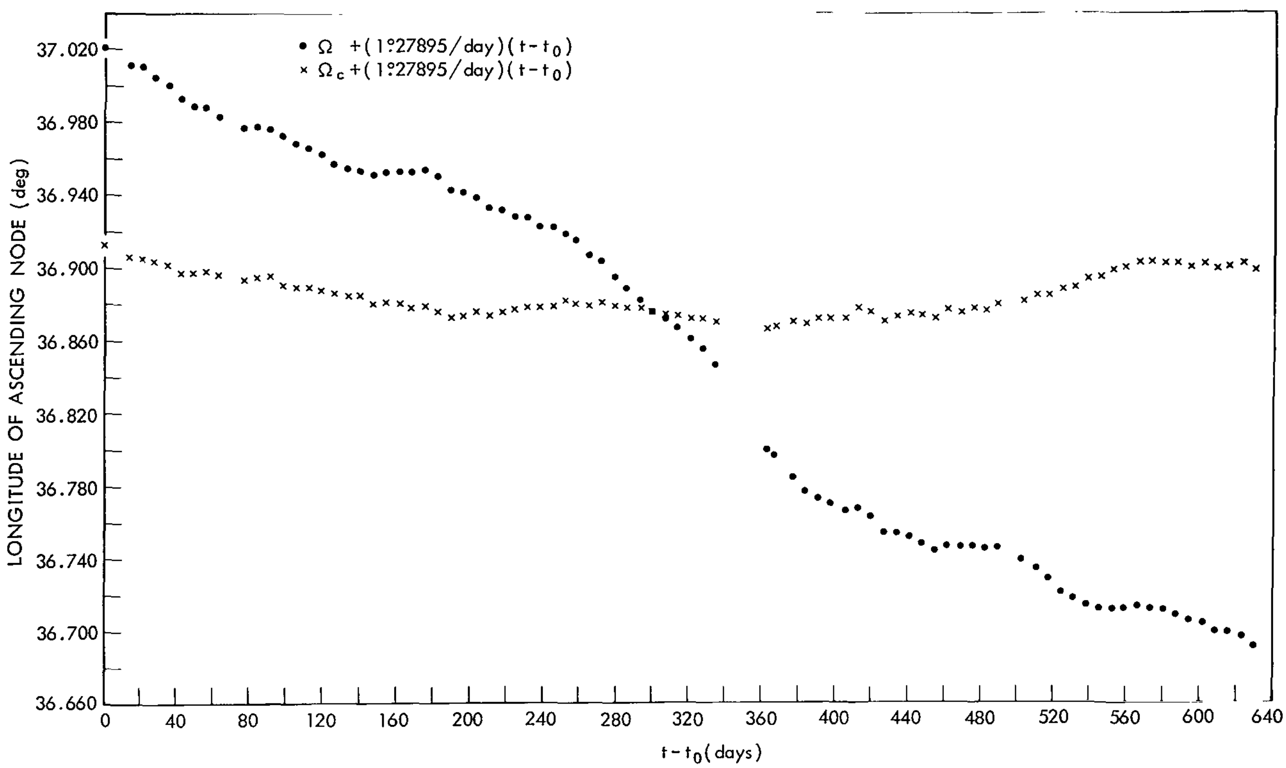

Figure 9—Longitude of ascending node for Relay 1.

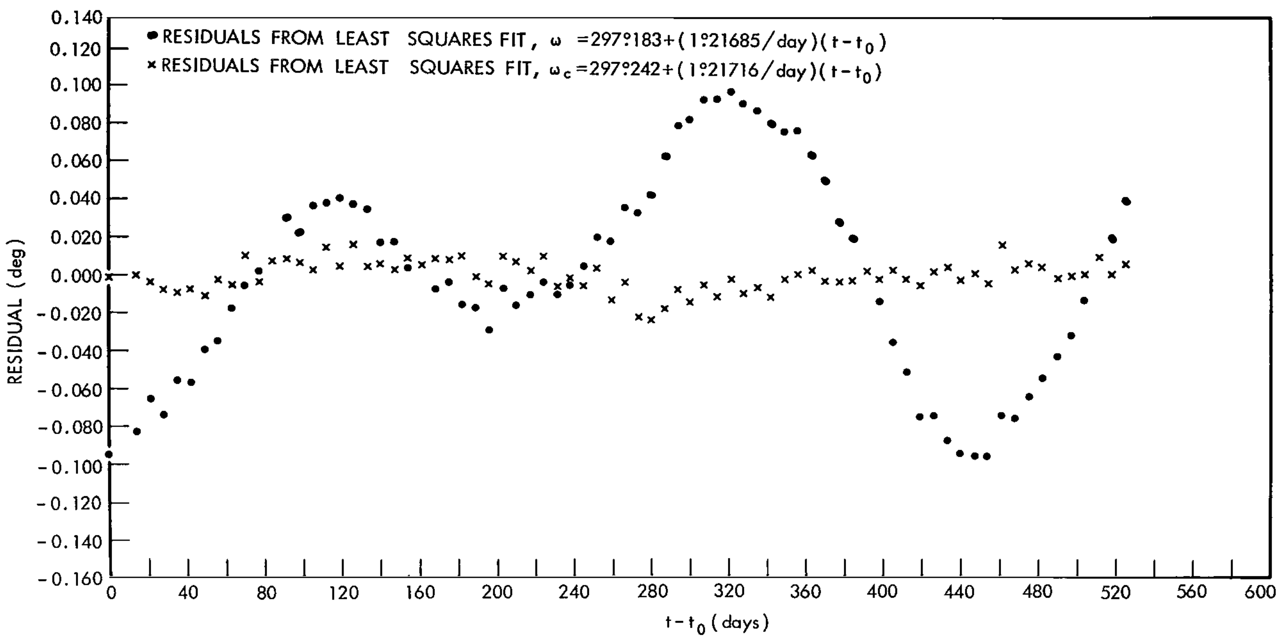

Figure 10—Residuals from least squares fits to argument of perigee for Telstar 2.

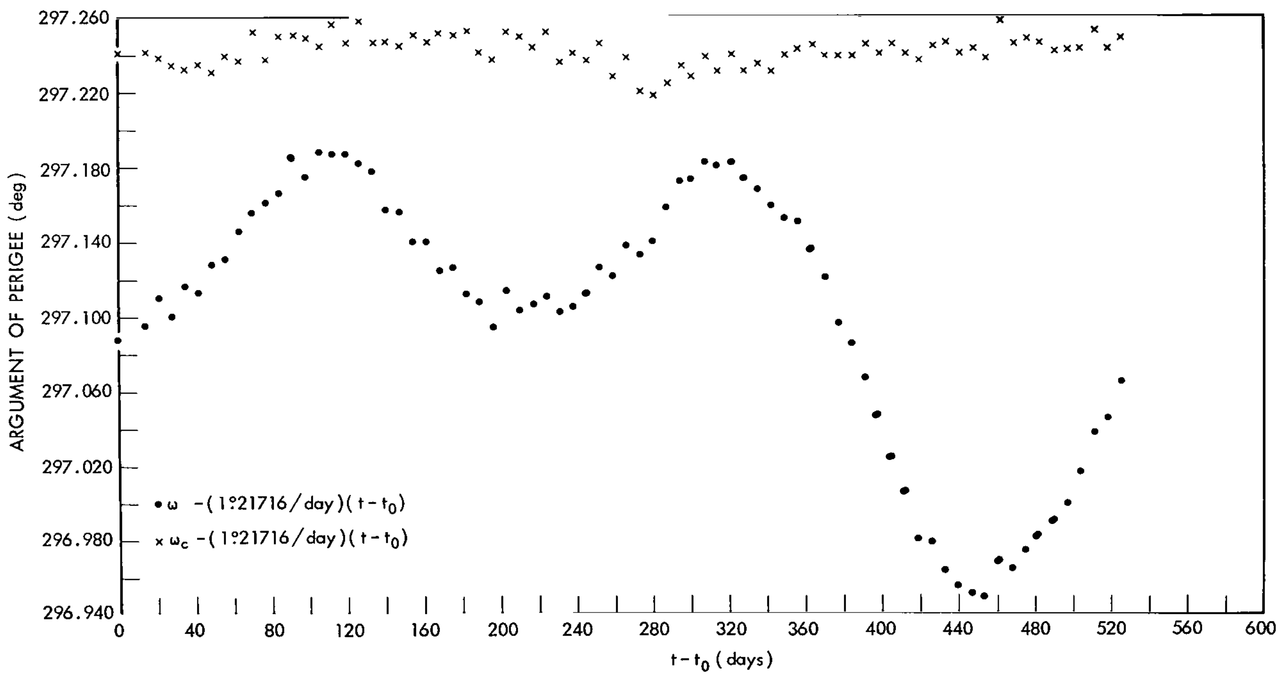

Figure 11—Argument of perigee for Telstar 2.

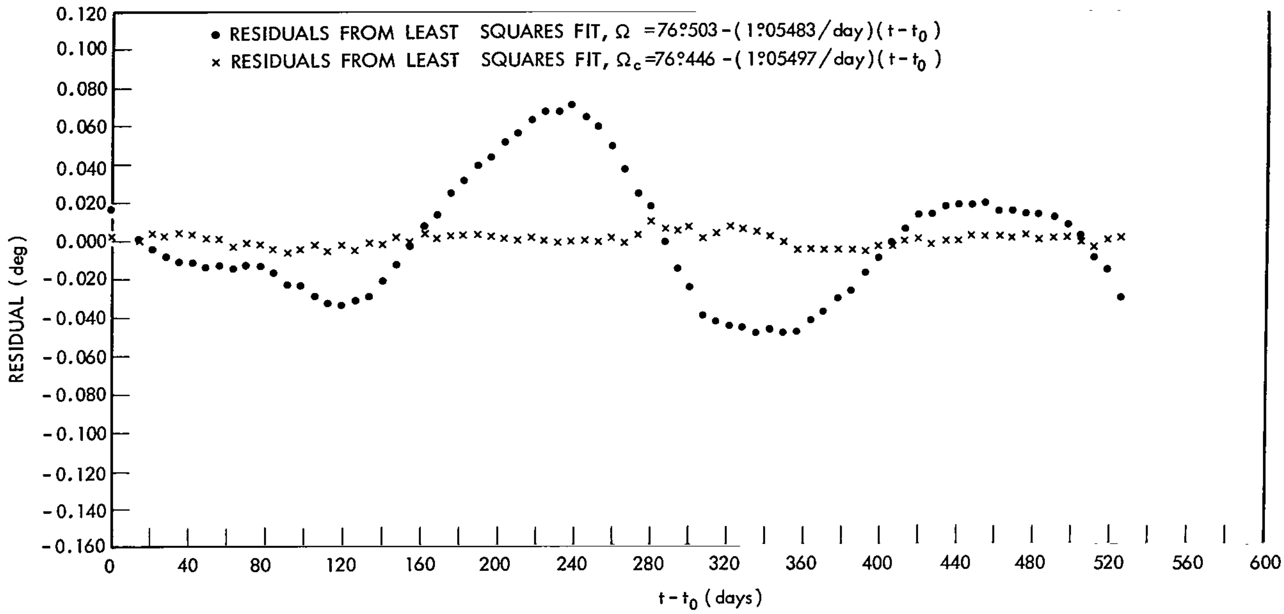

Figure 12—Residuals for least squares fits to longitude of ascending node for Telstar 2.

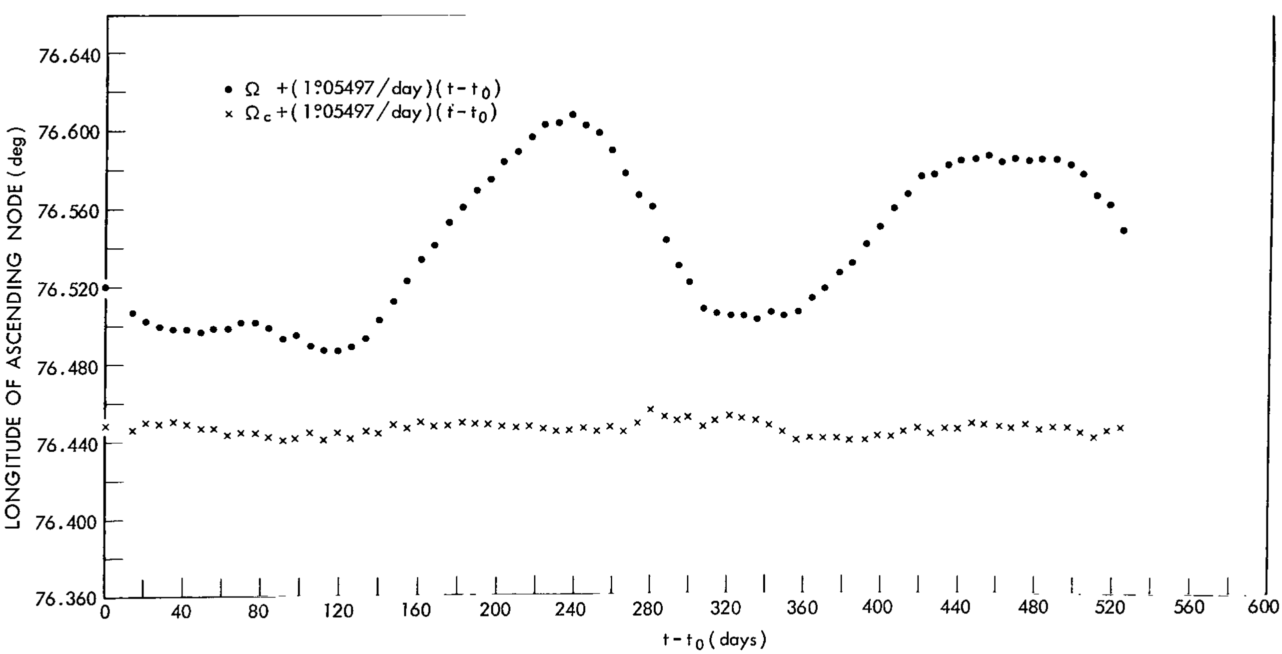

Figure 13—Longitude of ascending node for Telstar 2.

REFERENCES

1. Kozai, Y., "On the Effects of the Sun and Moon Upon the Motion of a Close Earth Satellite," SAO Special Rept. 22, Smithsonian Institution Astrophysical Observatory, March 20, 1959.

2. Musen, P., Bailie, A., and Upton, E., "Development of the Lunar and Solar Perturbations in the Motion of an Artificial Satellite," NASA Technical Note D-494, January 1961.

3. Kaula, W. M., "A Development of the Lunar and Solar Disturbing Functions for a Close Satellite," NASA Technical Note D-1126, January 1962.

4. Musen, P., "The Influence of the Solar Radiation Pressure on the Motion of an Artificial Satellite," *J. Geophys. Res.*, 65(5):1391-1396, May 1960.

5. Brouwer, D., "Solution of the Problem of Artificial Satellite Theory Without Drag," *Astron. J.*, 64(9):378-397, November 1959.

6. "The American Ephemeris and Nautical Almanac for the Year 1963," U. S. Government Printing Office: Washington, D. C., 1958.

7. "Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Alamanac," Her Majesty's Stationery Office: London, 1961.

Appendix A

Symbols

A	Effective presentation area of satellite, approximately $(1/4) \times$ total surface area
a	Semimajor axis of satellite's orbit
a'	Semimajor axis of disturbing body's orbit relative to earth
a_e	Mean equatorial radius of earth (6378.388 km.*)
e	Eccentricity of satellite's orbit
e'	Eccentricity of disturbing body's orbit relative to earth
e_c	Eccentricity corrected for long-period lunar and solar effects
F	Solar radiation pressure force constant
G	Gravitational constant
i	Inclination of satellite's orbital plane to earth's equatorial plane
i'	Inclination of disturbing body's orbital plane to earth's equatorial plane
i''	Inclination of lunar orbital plane to ecliptic
i_c	Inclination corrected for long-period lunar and solar effects
J_2, J_3, J_4, J_5	Zonal harmonic coefficients in earth's gravitational potential ($J_2 = 1.08219 \times 10^{-3}$, $J_3 = -2.285 \times 10^{-6}$, $J_4 = -2.123 \times 10^{-6}$, and $J_5 = -2.32 \times 10^{-7}$)*
ℓ	Mean anomaly of satellite
$\dot{\ell}$	Mean motion of mean anomaly
m	Mass of satellite
m'	Ratio of mass of disturbing body to mass of disturbing body plus mass of earth
n	Mean motion of satellite, $\approx a^{-3/2}$

*Values currently in use at Goddard Space Flight Center.

n'	Mean motion of disturbing body relative to earth
$P_2, P_3, \cdots$	Legendre polynomials
R	Force function for disturbing body
R_p	Solar radiation pressure disturbing function
r	Geocentric distance of satellite
r'	Geocentric distance of disturbing body
s	$\cos(\vec{r}, \vec{r}\,')$
T	Number of Julian Centuries (of 36525 days) from January 0.5, 1900
$t - t_0$	Number of days from initial epoch.
Γ''	Mean longitude of lunar perigee, measured in ecliptic from mean equinox of date to mean ascending node of lunar orbit, and then along the orbit
λ'	Mean longitude of disturbing body, measured in earth's equatorial plane from mean equinox of date to mean ascending node of disturbing body's orbit, and then along the orbit
$\dot{\lambda}'$	Mean secular motion of λ'
Ω	Longitude of ascending node of satellite's orbit
Ω'	Longitude of mean ascending node of disturbing body's orbit on earth's equator, measured from mean equinox of date
Ω''	Longitude of mean ascending node of lunar orbit on ecliptic, measured from mean equinox of date
$\dot{\Omega}$	Mean motion of longitude of ascending node
$\dot{\Omega}'$	Mean secular motion of Ω'
Ω_c	Longitude of ascending node corrected for long-period lunar and solar effects
ω	Argument of perigee of satellite's orbit
ω'	Argument of perigee of disturbing body's orbit, measured from mean ascending node on earth's equator
$\dot{\omega}$	Mean motion of the argument of perigee
$\dot{\omega}'$	Mean secular motion of ω'
ω_c	Argument of perigee corrected for long-period lunar and solar effects
☾	Mean longitude of moon, measured in ecliptic from mean equinox of date to mean ascending node of lunar orbit, and then along the orbit

The quantities $e_{\odot}$, $\lambda_{☾}$, etc. refer to the corresponding primed quantities when the disturbing body is the sun ($\odot$) or the moon (☾).

Appendix B

Tables

Table B1

Parameters in the Secular and Long-Period Disturbing Functions.

a (earth radii)	J_2/a^3	J_2^2/a^5	J_3/a^4	J_4/a^5	J_5/a^6	$m_☾ n_☾^2 a^2$	$m_\odot n_\odot^2 a^2$	$-Fa$
1.00	10.082×10^{-4}	11.70×10^{-7}	-22.85×10^{-7}	-21.23×10^{-7}	-232.0×10^{-9}	0.5507×10^{-7}	2.579×10^{-8}	3.537×10^{-9}
1.05	9.347	9.173	-18.81	-16.64	-173.1	.6071	2.843	3.714
1.10	8.129	7.269	-15.62	-13.18	-131.0	.6663	3.121	3.891
1.15	7.114	5.820	-13.06	-10.56	-100.3	.7283	3.411	4.068
1.20	6.262	4.705	-11.02	-8.535	-77.72	.7930	3.714	4.244
1.25	5.540	3.836	-9.360	-6.957	-60.82	.8605	4.030	4.421
1.30	4.925	3.153	-8.000	-5.718	-48.06	.9307	4.359	4.598
1.35	4.398	2.611	-6.879	-4.735	-38.33	1.004	4.700	4.775
1.40	3.943	2.176	-5.948	-3.947	-30.81	1.079	5.055	4.952
1.45	3.549	1.826	-5.169	-3.312	-24.96	1.158	5.422	5.129
1.5	3.206	1.542	-4.513	-2.796	-20.37	1.239	5.803	5.306
1.6	2.642	1.117	-3.487	-2.024	-13.83	1.410	6.602	5.659
1.7	2.202	.8244	-2.736	-1.495	-9.611	1.592	7.453	6.013
1.8	1.855	.6196	-2.177	-1.123	-6.822	1.784	8.356	6.367
1.9	1.577	.4727	-1.753	-.8574	-4.932	1.988	9.310	6.720
2.0	1.352	.3657	-1.428	-.6635	-3.625	2.203	10.32	7.074
2.2	1.016	.2272	-.9755	-.4120	-2.046	2.665	12.48	7.781
2.4	.7827	.1470	-.6888	-.2666	-1.214	3.172	14.86	8.489
2.6	.6156	.09856	-.5000	-.1787	-.7510	3.723	17.43	9.196
2.8	.4929	.06801	-.3718	-.1234	-.4814	4.317	20.22	9.904
3.0	.4007	.04816	-.2821	-.08737	-.3182	4.956	23.21	10.61
3.5	.2524	.02229	-.1523	-.04042	-.1262	6.746	31.59	12.38
4.0	.1691	.01143	-.08925	-.02073	-.05665	8.811	41.26	14.15
4.5	.1187	.006342	-.05572	-.00151	-.02794	11.15	52.22	15.92
5.0	.08656	.003746	-.03656	-.006794	-.01485	13.77	64.48	17.69

Table B2

Eccentricity and Inclination of Relay 1.
[a ≈ 1.6867 earth radii]

$t - t_0$*	e	e_c	i (deg)	i_c (deg)	$t - t_0$	e	e_c	i (deg)	i_c (deg)
0	0.28487	0.28430	47.524	47.509	314	0.28500	0.28427	47.527	47.508
14	.28478	.28429	47.523	47.507	321	.28500	.28423	47.525	47.509
21	.28474	.28425	47.523	47.508	328	.28505	.28425	47.522	47.507
28	.28474	.28430	47.524	47.510	335	.28505	.28424	47.520	47.509
35	.28468	.28425	47.521	47.509	363	.28496	.28419	47.511	47.508
42	.28466	.28428	47.518	47.506	367	.28495	.28423	47.510	47.508
49	.28462	.28425	47.518	47.509	377	.28487	.28421	47.510	47.508
56	.28463	.28429	47.518	47.508	384	.28482	.28422	47.509	47.509
63	.28455	.28423	47.517	47.509	391	.28473	.28422	47.509	47.508
77	.28449	.28424	47.515	47.509	398	.28467	.28422	47.508	47.510
84	.28449	.28427	47.513	47.510	406	.28457	.28423	47.507	47.510
91	.28437	.28418	47.509	47.506	413	.28452	.28422	47.506	47.510
98	.28445	.28429	47.509	47.510	420	.28446	.28423	47.502	47.510
105	.28439	.28428	47.507	47.509	427	.28443	.28421	47.502	47.509
112	.28439	.28429	47.506	47.510	434	.28439	.28424	47.501	47.512
119	.28436	.28429	47.506	47.510	441	.28437	.28422	47.504	47.510
126	.28435	.28428	47.507	47.511	448	.28436	.28423	47.505	47.512
133	.28433	.28429	47.507	47.510	455	.28434	.28420	47.507	47.510
140	.28434	.28429	47.509	47.509	462	.28435	.28421	47.508	47.511
147	.28435	.28432	47.509	47.509	469	.28433	.28417	47.509	47.509
154	.28434	.28428	47.512	47.508	476	.28436	.28417	47.511	47.510
161	.28433	.28428	47.510	47.509	483	.28437	.28414	47.512	47.510
168	.28437	.28428	47.510	47.508	490	.28443	.28416	47.513	47.510
175	.28435	.28426	47.509	47.511	504	.28450	.28415	47.510	47.509
182	.28437	.28422	47.506	47.508	511	.28449	.28411	47.509	47.510
189	.28440	.28425	47.505	47.510	518	.28454	.28412	47.508	47.507
196	.28441	.28420	47.504	47.510	525	.28452	.28411	47.511	47.510
203	.28442	.28422	47.505	47.510	532	.28450	.28407	47.512	47.508
210	.28444	.28418	47.506	47.512	539	.28452	.28412	47.516	47.510
217	.28449	.28421	47.507	47.509	546	.28454	.28412	47.518	47.509
224	.28452	.28419	47.509	47.510	553	.28448	.28413	47.520	47.508
231	.28457	.28422	47.513	47.510	560	.28451	.28416	47.523	47.510
238	.28460	.28420	47.515	47.511	567	.28444	.28414	47.524	47.509
245	.28465	.28421	47.516	47.510	574	.28443	.28412	47.524	47.509
252	.28466	.28419	47.517	47.510	581	.28443	.28417	47.523	47.508
258	.28473	.28425	47.517	47.509	588	.28446	.28417	47.523	47.508
265	.28475	.28421	47.518	47.509	595	.28445	.28419	47.522	47.508
272	.28481	.28426	47.519	47.509	602	.28451	.28418	47.521	47.507
279	.28480	.28421	47.521	47.509	609	.28453	.28420	47.521	47.508
286	.28486	.28426	47.522	47.509	616	.28460	.28419	47.521	47.508
293	.28486	.28422	47.524	47.508	623	.28466	.28422	47.520	47.508
300	.28494	.28428	47.526	47.509	630	.28474	.28420	47.517	47.507
307	.28493	.28423	47.528	47.510					

*$t - t_0$ is number of days since 5/4/63, 23 hr, 50 min UT.

Table B3

Argument of Perigee (Relay 1).

$t - t_0$	ω (deg)	Residuals* (deg)	ω_c (deg)	Residuals** (deg)	ω - (1°21206) $(t - t_0)$ (deg)	ω_c - (1°21206) $(t - t_0)$ (deg)
0	349.751	0.047	349.836	-0.014	349.751	349.836
14	6.732	.051	6.805	-.014	.763	.836
21	15.220	.050	15.278	-.025	.767	.825
28	23.710	.052	23.775	-.013	.772	.838
35	32.199	.052	32.255	-.017	.777	.833
42	40.666	.031	40.733	-.024	.759	.826
49	49.154	.030	49.215	-.026	.763	.824
56	57.635	.023	57.713	-.013	.760	.837
63	66.116	.015	66.193	-.017	.756	.833
77	83.066	-.012	83.164	-.015	.737	.835
84	91.543	-.023	91.659	-.004	.730	.846
91	100.029	-.026	100.145	-.002	.731	.848
98	108.501	-.042	108.631	-.001	.719	.849
105	116.995	-.037	117.119	.002	.729	.853
112	125.477	-.043	125.608	.007	.726	.857
119	133.967	-.041	134.088	.003	.732	.853
126	142.460	-.037	142.582	.012	.740	.862
133	150.949	-.036	151.055	.001	.745	.851
140	159.439	-.035	159.539	.000	.750	.851
147	167.941	-.021	168.026	.003	.768	.853
154	176.438	-.013	176.514	.006	.780	.857
161	184.932	-.007	184.992	.000	.790	.850
168	193.440	.012	193.486	.009	.814	.860
175	201.924	.008	201.960	-.001	.813	.850
182	210.421	.017	210.444	-.001	.826	.849
189	218.916	.022	218.932	.003	.836	.853
196	227.408	.026	227.412	-.002	.844	.848
203	235.894	.024	235.898	-.001	.845	.850
210	244.383	.024	244.379	-.005	.850	.846
217	252.867	.020	252.870	.002	.850	.853
224	261.353	.017	261.352	-.000	.851	.850
231	269.826	.002	269.838	.002	.840	.852
238	278.308	-.004	278.321	-.000	.838	.850
245	286.783	-.018	286.814	.008	.828	.858
252	295.263	-.027	295.293	.003	.823	.853
258	302.526	-.039	302.571	.008	.814	.859
265	311.013	-.041	311.054	.007	.817	.858
272	319.490	-.052	319.544	.013	.810	.863
279	327.983	-.048	328.026	.011	.818	.861
286	336.469	-.051	336.519	.019	.819	.869
293	344.967	-.041	345.002	.018	.833	.868
300	353.455	-.041	353.494	.025	.837	.875

*From least squares fit, $\omega = 349.°704 + (1.°21264/\text{day})(t - t_0)$.

**From least squares fit, $\omega_c = 349.°850 + (1.°21206/\text{day})(t - t_0)$.

Table B3 - Continued

Argument of Perigee (Relay 1) - Continued.

$t - t_0$	ω (deg)	Residuals* (deg)	ω_c (deg)	Residuals** (deg)	ω - (1°21206) $(t - t_0)$ (deg)	ω_c - (1°21206) $(t - t_0)$ (deg)
307	1.949	-0.036	1.969	0.015	349.846	349.866
314	10.436	-.037	10.457	.020	.849	.870
321	18.937	-.025	18.937	.015	.865	.865
328	27.421	-.029	27.421	.014	.865	.864
335	35.919	-.020	35.899	.008	.878	.858
363	69.898	.006	69.842	.014	.920	.864
367	74.751	.008	74.696	.019	.924	.870
377	86.870	.001	86.804	.006	.923	.856
384	95.363	.005	95.301	.019	.931	.869
391	103.869	.022	103.798	.032	.953	.882
398	112.338	.003	112.271	.020	.937	.870
406	122.041	.005	121.967	.019	.944	.870
413	130.530	.005	130.453	.022	.948	.872
420	139.007	-.006	138.925	.009	.941	.859
427	147.499	-.002	147.414	.013	.949	.863
434	155.983	-.007	155.889	.004	.948	.854
441	164.478	-.000	164.374	.004	.959	.855
448	172.965	-.002	172.855	.001	.961	.851
455	181.466	.010	181.342	.004	.978	.854
462	189.955	.011	189.824	.001	.982	.851
469	198.454	.022	198.303	-.005	.997	.846
476	206.945	.024	206.789	-.003	350.004	.847
483	215.448	.038	215.270	-.006	.022	.844
490	223.943	.045	223.759	-.001	.033	.849
504	240.939	.064	240.730	.001	.060	.851
511	249.431	.068	249.204	-.010	.068	.840
518	257.925	.074	257.698	-.001	.077	.850
525	266.415	.074	266.173	-.010	.082	.840
532	274.906	.077	274.669	.002	.089	.853
539	283.384	.066	283.139	-.013	.082	.838
546	291.863	.057	291.631	-.005	.077	.845
553	300.337	.043	300.104	-.017	.067	.834
560	308.808	.026	308.592	-.013	.054	.837
567	317.286	.015	317.071	-.018	.047	.832
574	325.760	.000	325.561	-.012	.037	.838
581	334.233	-.015	334.038	-.020	.025	.830
588	342.714	-.023	342.534	-.008	.022	.842
595	351.188	-.037	351.008	-.019	.011	.831
602	359.666	-.047	359.498	-.013	.005	.837
609	8.149	-.053	7.983	-.013	.004	.837
616	16.625	-.066	16.469	-.012	349.995	.839
623	25.106	-.073	24.949	-.016	.992	.834
630	33.595	-.072	33.445	-.004	.996	.846

*From least squares fit, $\omega = 349.^\circ 704 + (1.^\circ 21264/\text{day})(t - t_0)$.

**From least squares fit, $\omega_c = 349.^\circ 850 + (1.^\circ 21206/\text{day})(t - t_0)$.

Table B4

Longitude of Ascending Node (Relay 1).

$t-t_0$	Ω (deg)	Residuals* (deg)	Ω_c (deg)	Residuals** (deg)	$\Omega + (1.27895)(t-t_0)$ (deg)	$\Omega_c + (1.27895)(t-t_0)$ (deg)
0	37.021	-0.014	36.913	0.028	37.021	36.913
14	19.106	-.015	19.001	.021	.011	.906
21	10.152	-.012	10.048	.021	.010	.906
28	1.194	-.014	1.093	.019	.005	.904
35	352.237	-.014	352.138	.016	.000	.901
42	343.277	-.017	343.181	.012	36.993	.897
49	334.320	-.017	334.228	.012	.989	.897
56	325.367	-.014	325.277	.013	.988	.898
63	316.409	-.015	316.323	.012	.983	.897
77	298.497	-.014	298.414	.009	.976	.894
84	289.547	-.007	289.463	.010	.979	.895
91	280.592	-.005	280.510	.010	.977	.895
98	271.635	-.005	271.553	.005	.973	.890
105	262.678	-.006	262.599	.004	.968	.889
112	253.723	-.004	253.647	.005	.966	.890
119	244.767	-.003	244.693	.003	.963	.888
126	235.809	-.004	235.738	.001	.957	.886
133	226.854	-.003	226.783	-.001	.955	.884
140	217.900	.000	217.831	.000	.954	.885
147	208.945	.002	208.874	-.004	.951	.881
154	199.994	.008	199.923	-.003	.953	.882
161	191.042	.012	190.969	-.004	.954	.881
168	182.089	.016	182.014	-.007	.953	.878
175	173.137	.021	173.063	-.005	.954	.880
182	164.181	.022	164.106	-.009	.951	.876
189	155.222	.019	155.151	-.011	.944	.874
196	146.267	.021	146.199	-.010	.942	.875
203	137.312	.023	137.249	-.008	.940	.877
210	128.353	.020	128.294	-.011	.933	.874
217	119.399	.023	119.344	-.008	.932	.877
224	110.443	.024	110.392	-.007	.929	.878
231	101.490	.028	101.440	-.006	.929	.879
238	92.532	.026	92.487	-.006	.923	.879
245	83.579	.030	83.536	-.005	.923	.880
252	74.623	.031	74.586	-.003	.920	.882
258	66.946	.031	66.910	-.004	.916	.881
265	57.985	.027	57.957	-.005	.908	.880
272	49.028	.027	49.005	-.004	.904	.881
279	40.067	.022	40.051	-.005	.896	.880
286	31.108	.020	31.097	-.007	.889	.878
293	22.150	.019	22.144	-.007	.884	.878
300	13.190	.016	13.190	-.009	.877	.876

*From least squares fit, $\Omega = 37.^\circ 035 - (1.^\circ 27953/\text{day})(t - t_0)$.

**From least squares fit, $\Omega_c = 36.^\circ 885 - (1.^\circ 27895/\text{day})(t - t_0)$.

Table B4 - Continued

Longitude of Ascending Node (Relay 1) - Continued.

$t - t_0$	Ω (deg)	Residuals* (deg)	Ω_c (deg)	Residuals** (deg)	$\Omega + (1.27895)(t - t_0)$ (deg)	$\Omega_c + (1.27895)(t - t_0)$ (deg)
307	4.234	0.016	4.236	-0.010	36.873	36.875
314	355.276	.014	355.282	-.011	.867	.874
321	346.317	.013	346.328	-.013	.862	.872
328	337.359	.012	337.376	-.012	.856	.873
335	328.397	.006	328.422	-.014	.847	.871
363	292.540	-.024	292.607	-.018	.801	.867
367	287.421	-.025	287.492	-.017	.797	.868
377	274.619	-.032	274.705	-.014	.785	.871
384	265.659	-.035	265.751	-.016	.777	.869
391	256.702	-.035	256.801	-.013	.773	.872
398	247.747	-.034	247.849	-.012	.770	.873
406	237.511	-.033	237.617	-.012	.767	.873
413	228.560	-.027	228.671	-.006	.768	.879
420	219.603	-.028	219.715	-.009	.764	.876
427	210.641	-.033	210.758	-.013	.755	.872
434	201.688	-.029	201.807	-.011	.754	.874
441	192.733	-.027	192.857	-.009	.752	.876
448	183.777	-.027	183.903	-.010	.748	.875
455	174.820	-.027	174.948	-.012	.745	.873
462	165.871	-.020	166.001	-.007	.748	.878
469	156.917	-.017	157.046	-.009	.746	.876
476	147.965	-.012	148.096	-.007	.747	.878
483	139.011	-.009	139.142	-.008	.746	.877
490	130.059	-.005	130.192	-.005	.747	.880
504	112.147	-.003	112.289	-.003	.740	.882
511	103.189	-.004	103.340	.001	.735	.886
518	94.231	-.006	94.387	.001	.729	.886
525	85.271	-.009	85.438	.004	.722	.889
532	76.315	-.008	76.486	.005	.719	.890
539	67.358	-.008	67.538	.009	.715	.894
546	58.404	-.006	58.586	.011	.713	.896
553	49.451	-.002	49.638	.014	.713	.899
560	40.498	.002	40.686	.016	.713	.901
567	31.547	.008	31.737	.019	.715	.904
574	22.593	.010	22.784	.019	.713	.904
581	13.639	.014	13.831	.018	.712	.903
588	4.684	.015	4.877	.018	.710	.903
595	355.729	.016	355.923	.016	.707	.901
602	346.774	.019	346.971	.017	.705	.902
609	337.817	.018	338.016	.015	.700	.900
616	328.864	.022	329.065	.016	.700	.901
623	319.909	.023	320.114	.017	.698	.902
630	310.951	.022	311.157	.014	.692	.899

*From least squares fit, $\Omega = 37.^\circ035 - (1.^\circ27953/\text{day})(t - t_0)$.
**From least squares fit, $\Omega_c = 36.^\circ885 - (1.^\circ27895/\text{day})(t - t_0)$.

Table B5

Eccentricity and Inclination of Telstar 2.
[a ≈ 1.9232 earth radii]

$t - t_0$ *	e	e_c	i (deg)	i_c (deg)
0	0.40122	0.40114	42.753	42.757
14	.40129	.40114	42.759	42.759
21	.40135	.40113	42.762	42.756
28	.40136	.40114	42.765	42.756
35	.40140	.40115	42.770	42.755
42	.40138	.40116	42.772	42.755
49	.40140	.40114	42.776	42.754
56	.40139	.40115	42.776	42.753
63	.40142	.40116	42.777	42.754
70	.40138	.40116	42.777	42.755
77	.40138	.40114	42.773	42.755
84	.40136	.40116	42.772	42.756
91	.40134	.40115	42.769	42.757
98	.40128	.40117	42.766	42.756
105	.40124	.40116	42.764	42.757
112	.40115	.40116	42.763	42.757
119	.40112	.40119	42.761	42.757
126	.40097	.40117	42.761	42.757
133	.40090	.40118	42.760	42.757
140	.40077	.40116	42.757	42.757
147	.40069	.40116	42.756	42.757
154	.40058	.40115	42.752	42.758
161	.40053	.40117	42.750	42.756
168	.40047	.40115	42.745	42.756
175	.40047	.40117	42.743	42.755
182	.40044	.40115	42.742	42.756
189	.40046	.40115	42.743	42.758
196	.40046	.40109	42.743	42.754
203	.40058	.40115	42.745	42.756
210	.40065	.40112	42.748	42.755
217	.40074	.40111	42.749	42.758
224	.40091	.40113	42.750	42.754
231	.40097	.40109	42.745	42.754
238	.40116	.40112	42.748	42.756
245	.40125	.40112	42.746	42.759
252	.40140	.40111	42.745	42.759
259	.40149	.40115	42.740	42.757
266	.40159	.40113	42.736	42.755

$t - t_0$	e	e_c	i (deg)	i_c (deg)
273	0.40162	0.40115	42.739	42.757
280	.40166	.40111	42.739	42.759
287	.40162	.40112	42.743	42.758
294	.40164	.40111	42.745	42.758
300	.40157	.40112	42.750	42.757
307	.40157	.40113	42.753	42.758
314	.40147	.40115	42.759	42.757
321	.40142	.40113	42.768	42.763
328	.40128	.40116	42.768	42.759
335	.40124	.40115	42.770	42.758
342	.40111	.40117	42.772	42.759
349	.40106	.40114	42.772	42.757
356	.40095	.40118	42.771	42.756
363	.40095	.40115	42.774	42.756
370	.40087	.40118	42.774	42.757
377	.40090	.40114	42.776	42.756
384	.40087	.40120	42.778	42.758
391	.40094	.40117	42.780	42.757
398	.40091	.40119	42.780	42.758
405	.40100	.40116	42.781	42.758
412	.40098	.40119	42.780	42.757
419	.40108	.40116	42.779	42.758
426	.40106	.40118	42.777	42.756
433	.40115	.40115	42.776	42.758
440	.40114	.40119	42.775	42.757
447	.40124	.40117	42.774	42.757
454	.40124	.40120	42.773	42.757
461	.40133	.40117	42.773	42.756
468	.40135	.40121	42.772	42.757
475	.40144	.40117	42.774	42.756
482	.40145	.40118	42.771	42.757
490	.40156	.40113	42.771	42.758
497	.40159	.40115	42.766	42.759
504	.40170	.40113	42.762	42.759
511	.40175	.40117	42.755	42.759
518	.40181	.40111	42.751	42.761
525	.40186	.40114	42.745	42.759

*$t - t_0$ is number of days since 8/18/63, 11 hrs., 55 min. U.T.

Table B6

Argument of Perigee (Telstar 2).

$t - t_0$	ω (deg)	Residuals* (deg)	ω_c (deg)	Residuals** (deg)	ω - (1°21716) $(t - t_0)$ (deg)	ω_c - (1°21716) $(t - t_0)$ (deg)
0	297.088	-0.095	297.241	-0.001	297.088	297.241
14	314.136	-.083	314.282	-.000	.096	.242
21	322.671	-.066	322.798	-.004	.111	.238
28	331.181	-.074	331.315	-.007	.101	.235
35	339.717	-.056	339.833	-.010	.116	.233
42	348.234	-.057	348.356	-.007	.113	.235
49	356.769	-.040	356.871	-.012	.128	.230
56	5.292	-.035	5.400	-.003	.131	.239
63	13.827	-.018	13.918	-.005	.146	.237
70	22.357	-.006	22.453	.010	.156	.252
77	30.883	.002	30.960	-.004	.162	.239
84	39.408	.009	39.491	.008	.167	.250
91	47.947	.030	48.012	.009	.185	.251
98	56.457	.023	56.531	.007	.176	.249
105	64.990	.037	65.047	.003	.188	.245
112	73.509	.039	73.579	.015	.187	.257
119	82.030	.041	82.089	.005	.188	.247
126	90.544	.038	90.620	.017	.183	.259
133	99.060	.036	99.129	.005	.178	.247
140	107.560	.018	107.650	.006	.158	.248
147	116.078	.018	116.168	.004	.156	.246
154	124.582	.004	124.694	.009	.140	.251
161	133.103	.006	133.210	.006	.140	.248
168	141.608	-.007	141.734	.010	.125	.252
175	150.129	-.003	150.254	.009	.126	.251
182	158.635	-.015	158.776	.011	.112	.253
189	167.151	-.017	167.284	-.001	.108	.242
196	175.657	-.029	175.801	-.004	.094	.238
203	184.197	-.007	184.336	.011	.114	.253
210	192.707	-.015	192.853	.008	.104	.250
217	201.230	-.010	201.368	.003	.107	.245
224	209.755	-.003	209.896	.010	.111	.252
231	218.266	-.010	218.399	-.006	.102	.236
238	226.789	-.004	226.924	-.001	.106	.241
245	235.317	.005	235.441	-.005	.113	.237
252	243.850	.021	243.970	.004	.127	.246
259	252.366	.018	252.473	-.013	.122	.229
266	260.902	.036	261.003	-.003	.138	.239

*From least squares fit, $\omega = 297.^{\circ}183 + (1.^{\circ}21685/\text{day})(t - t_0)$.

**From least squares fit, $\omega_c = 297.^{\circ}242 + (1.^{\circ}21716/\text{day})(t - t_0)$.

Table B6 - Continued

Argument of Perigee (Telstar 2) - Continued.

$t-t_0$	ω (deg)	Residuals* (deg)	ω_c (deg)	Residuals** (deg)	ω - (1.°21716) $(t-t_0)$ (deg)	ω_c - (1.°21716) $(t-t_0)$ (deg)
273	269.417	0.034	269.505	-0.021	297.133	297.221
280	277.945	.043	278.023	-.023	.141	.219
287	286.483	.064	286.549	-.017	.159	.225
294	295.017	.080	295.078	-.008	.173	.234
300	302.321	.083	302.375	-.014	.174	.228
307	310.850	.094	310.905	-.004	.183	.238
314	319.368	.094	319.419	-.011	.181	.231
321	327.890	.098	327.948	-.002	.183	.240
328	336.402	.092	336.460	-.010	.174	.232
335	344.916	.088	344.984	-.006	.168	.236
342	353.427	.081	353.499	-.011	.160	.231
349	1.941	.077	2.028	-.002	.153	.240
356	10.459	.077	10.551	.001	.151	.244
363	18.964	.064	19.074	.004	.136	.246
370	27.469	.051	27.588	-.002	.121	.240
377	35.965	.029	36.108	-.002	.097	.240
384	44.474	.020	44.628	-.003	.086	.239
391	52.975	.004	53.154	.004	.067	.246
398	61.476	-.013	61.669	-.001	.048	.241
405	69.973	-.035	70.195	.004	.024	.246
412	78.475	-.051	78.709	-.002	.006	.241
419	86.969	-.075	87.226	-.005	296.980	.237
426	95.488	-.074	95.754	.003	.979	.245
433	103.993	-.087	104.276	.005	.964	.247
440	112.504	-.094	112.790	-.001	.955	.241
447	121.020	-.095	121.313	.002	.951	.244
454	129.538	-.095	129.828	-.004	.949	.238
461	138.078	-.074	138.368	.016	.968	.258
468	146.594	-.075	146.875	.004	.964	.246
475	155.124	-.063	155.399	.007	.974	.249
482	163.652	-.053	163.916	.004	.982	.246
490	173.398	-.042	173.649	-.000	.991	.242
497	181.927	-.031	182.169	.000	297.000	.242
504	190.464	-.012	190.690	.001	.017	.243
511	199.005	.011	199.220	.010	.037	.252
518	207.532	.021	207.730	.001	.045	.243
525	216.070	.041	216.256	.007	.063	.249

*From least squares fit, $\omega = 297.^\circ183 + (1.^\circ21685/\text{day})(t - t_0)$.
**From least squares fit, $\omega_c = 297.^\circ242 + (1.^\circ21716/\text{day})(t - t_0)$.

Table B7

Longitude of Ascending Node (Telstar 2).

$t - t_0$	Ω (deg)	Residuals* (deg)	Ω_c (deg)	Residuals** (deg)	$\Omega + (1.05497)(t - t_0)$ (deg)	$\Omega_c + (1.05497)(t - t_0)$ (deg)
0	76.520	0.017	76.449	0.003	76.520	76.449
14	61.737	.001	61.676	-.000	.507	.446
21	54.348	-.004	54.295	.004	.502	.450
28	46.960	-.008	46.910	.003	.499	.449
35	39.574	-.010	39.526	.004	.498	.450
42	32.189	-.012	32.141	.003	.498	.449
49	24.803	-.014	24.753	.000	.497	.447
56	17.420	-.013	17.368	.001	.499	.447
63	10.035	-.014	9.980	-.003	.498	.443
70	2.653	-.012	2.596	-.002	.501	.445
77	355.269	-.013	355.211	-.002	.502	.444
84	347.881	-.017	347.824	-.004	.499	.442
91	340.491	-.023	340.438	-.006	.494	.440
98	333.107	-.023	333.054	-.004	.494	.442
105	325.718	-.029	325.672	-.002	.490	.444
112	318.330	-.032	318.283	-.006	.487	.440
119	310.945	-.033	310.902	-.002	.487	.444
126	303.564	-.031	303.515	-.005	.491	.442
133	296.182	-.029	296.134	-.001	.494	.445
140	288.806	-.021	288.748	-.002	.503	.445
147	281.431	-.012	281.367	.002	.513	.448
154	274.057	-.003	273.980	-.000	.523	.446
161	266.684	.008	266.599	.004	.534	.450
168	259.306	.014	259.212	.001	.542	.448
175	251.932	.024	251.828	.002	.553	.449
182	244.556	.031	244.444	.003	.561	.450
189	237.180	.039	237.059	.003	.570	.449
196	229.801	.044	229.674	.003	.576	.449
203	222.424	.051	222.288	.002	.584	.448
210	215.045	.056	214.902	.000	.590	.447
217	207.668	.063	207.519	.002	.598	.448
224	200.289	.068	200.132	.000	.604	.447
231	192.906	.068	192.746	-.001	.605	.445
238	185.525	.071	185.362	-.001	.609	.446
245	178.135	.064	177.978	.000	.603	.447
252	170.746	.060	170.592	-.001	.599	.446
259	163.352	.050	163.210	.002	.590	.448
266	155.956	.037	155.822	-.001	.579	.445

*From least squares fit, $\Omega = 76.^\circ503 - (1.^\circ05483/\text{day})(t - t_0)$.
**From least squares fit, $\Omega_c = 76.^\circ446 - (1.^\circ05497/\text{day})(t - t_0)$.

Table B7 - Continued

Longitude of Ascending Node (Telstar 2) - Continued.

$t - t_0$	Ω (deg)	Residuals* (deg)	Ω_c (deg)	Residuals** (deg)	$\Omega + (1.05497)(t - t_0)$ (deg)	$\Omega_c + (1.05497)(t - t_0)$ (deg)
273	148.559	0.024	148.441	0.003	76.567	76.449
280	141.169	.018	141.064	.010	.562	.456
287	133.766	-.001	133.675	.006	.544	.452
294	126.369	-.015	126.289	.005	.531	.451
300	120.030	-.025	119.960	.006	.522	.453
307	112.631	-.039	112.571	.001	.508	.448
314	105.244	-.043	105.188	.004	.506	.450
321	97.858	-.045	97.806	.007	.506	.453
328	90.474	-.046	90.421	.006	.505	.452
335	83.087	-.048	83.035	.005	.503	.451
342	75.705	-.047	75.647	.002	.506	.448
349	68.320	-.048	68.259	-.001	.506	.445
356	60.936	-.048	60.870	-.006	.507	.441
363	53.558	-.042	53.486	-.004	.514	.442
370	46.179	-.038	46.101	-.005	.519	.441
377	38.802	-.030	38.716	-.005	.528	.441
384	31.422	-.027	31.331	-.006	.532	.441
391	24.047	-.018	23.945	-.006	.542	.440
398	16.672	-.009	16.564	-.003	.552	.443
405	9.296	-.001	9.178	-.004	.561	.442
412	1.919	.006	1.796	-.001	.569	.445
419	354.543	.013	354.413	.001	.577	.447
426	347.159	.013	347.025	-.002	.578	.444
433	339.779	.017	339.642	-.000	.583	.446
440	332.397	.018	332.257	-.000	.585	.446
447	325.013	.018	324.875	.003	.586	.449
454	317.630	.019	317.490	.002	.588	.448
461	310.241	.014	310.105	.002	.584	.448
468	302.858	.015	302.719	.000	.586	.447
475	295.473	.014	295.336	.002	.586	.448
482	288.089	.013	287.949	-.000	.586	.446
490	279.649	.012	279.510	.001	.586	.447
497	272.261	.008	272.125	.001	.583	.447
504	264.871	.002	264.738	-.002	.578	.445
511	257.476	-.010	257.350	-.004	.567	.442
518	250.086	-.016	249.969	-.001	.562	.446
525	242.688	-.030	242.586	.001	.549	.447

*From least squares fit, $\Omega = 76.^\circ 503 - (1.^\circ 05483/\text{day})(t - t_0)$.
**From least squares fit, $\Omega_c = 76.^\circ 446 - (1.^\circ 05497/\text{day})(t - t_0)$.

Table B8

Principal Long-Period Terms in Oblateness and Luni-Solar Perturbations in Eccentricity of Telstar 2.

Source	Amplitude	Argument	Period (days)
Oblateness (J_3/J_2)	373×10^{-6}	ω	296
Oblateness (J_4/J_2)	-60	2ω	148
Oblateness (J_2)	17	2ω	148
Oblateness (J_5/J_2)	5	ω	296
Oblateness (J_5/J_2)	-1	3ω	99
Lunar gravity	-220	$2(\Omega_{☾} - \omega - \Omega)$	1121
Lunar gravity	192	$\Omega_{☾} - 2\omega - \Omega$	261
Lunar gravity	-95	2ω	148
Lunar gravity	33	$2(\lambda_{☾} - \omega - \Omega)$	14
Lunar gravity	11	$2\lambda_{☾} - \Omega_{☾} - 2\omega - \Omega$	14
Lunar gravity	-11	$\Omega_{☾} + 2\omega - \Omega$	103
Solar gravity	245	$2(\lambda_{\odot} - \omega - \Omega)$	219
Solar gravity	221	$2\lambda_{\odot} - \Omega_{\odot} - 2\omega - \Omega$	607
Solar gravity	-106	$2(\Omega_{\odot} - \omega - \Omega)$	1111
Solar gravity	90	$\Omega_{\odot} - 2\omega - \Omega$	261
Solar gravity	-42	2ω	148
Solar gravity	-35	$2(\lambda_{\odot} - \Omega_{\odot} - \omega)$	781
Solar gravity	8	$\lambda_{\odot} - \omega_{\odot} - 2\omega - \Omega$	917
Solar gravity	-6	$\Omega_{\odot} + 2\omega - \Omega$	103
Solar radiation pressure	42	$\omega + \Omega - \lambda_{\odot}$	437
Solar radiation pressure	-24	$\omega - \lambda_{\odot}$	1562

Table B9

Principal Long-Period Terms in Oblateness and Luni-Solar Perturbations in Eccentricity of Relay 1.

Source	Amplitude	Argument	Period (days)
Oblateness (J_3/J_2)	520×10^{-6}	ω	298
Oblateness (J_4/J_2)	-71	2ω	149
Oblateness (J_2)	18	2ω	149
Oblateness (J_5/J_2)	7	ω	298
Oblateness (J_5/J_2)	7	3ω	99
Lunar gravity	304	$2(\Omega_{☾} - \omega - \Omega)$	2584
Lunar gravity	147	$\Omega_{☾} - 2\omega - \Omega$	315
Lunar gravity	-68	2ω	149
Lunar gravity	19	$2(\lambda_{☾} - \omega - \Omega)$	14
Lunar gravity	9	$2\lambda_{☾} - \Omega_{☾} - 2\omega - \Omega$	14
Lunar gravity	-7	$\Omega_{☾} + 2\omega - \Omega$	97
Solar gravity	149	$2(\Omega_{\odot} - \omega - \Omega)$	2691
Solar gravity	109	$2(\lambda_{\odot} - \omega - \Omega)$	171
Solar gravity	101	$2\lambda_{\odot} - \Omega_{\odot} - 2\omega - \Omega$	436
Solar gravity	70	$\Omega_{\odot} - 2\omega - \Omega$	314
Solar gravity	-30	2ω	149
Solar gravity	-30	$2(\lambda_{\odot} - \Omega_{\odot} - \omega)$	795
Solar gravity	12	$\lambda_{\odot} - \omega_{\odot} - 2\omega - \Omega$	2256
Solar radiation pressure	31	$\omega + \Omega - \lambda_{\odot}$	342
Solar radiation pressure	-26	$\omega - \lambda_{\odot}$	1590

NASA-Langley, 1966

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Technical information generated in connection with a NASA contract or grant and released under NASA auspices.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

TECHNICAL REPRINTS: Information derived from NASA activities and initially published in the form of journal articles.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities but not necessarily reporting the results of individual NASA-programmed scientific efforts. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546